Content for Mathematics AS and A level for teaching from 2017

Content required for AS mathematics is shown in bold text within square brackets. This, assessed in

		Underground Maths Review Questions	Review Questions chosen by the Underground Maths Team. Hosted by Nrich.	
A	**Proof**			
	Content			
A1	**[Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion]** **[Disproof by counter example]** Proof by contradiction (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar proofs)	When is 6×7=426×7=42 a counter-example?	Why is at most one of these numbers rational?	
B	**Algebra and functions**			
	Content			
B1	**[Understand and use the laws of indices for all rational exponents]**	Can we simplify these expressions involving indices?	When is this product of powers an integer?	Given these power facts, can we show $z^6=cx$?
B2	**[Use and manipulate surds, including rationalising the denominator]**	Can we write √2016+√56 as a power of 14?	Can we show this surd is less than 6?	
B3	**[Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown]**	Given the minimum point, what's this parabola's equation?		When do these simultaneous equations have no real solutions?
B4	**[Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation]**	Can we solve these simultaneous equations, one linear, one quadratic?	Can we solve these simultaneous equations of degree 1 and 2?	
B5	**[Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions]** **[Express solutions through correct use of 'and' and 'or', or through set notation]** **[Represent linear and quadratic inequalities such as $y > x+1$ and $y > ax^2+bx+c$ graphically]**	When is $12x^2+7x+7x-10$ negative?	When are these quadratic inequalities true together?	When is the inequality $x^4<8x^2+9$ satisfied?
B6	**[Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem]** Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only)	If dividing by x+ax+a or x−2ax−2a gives the same remainder, what's aa?	Can we factorise 6x3+5x2−17x−6 completely?	

B7	**[Understand and use graphs of functions; sketch curves defined by simple equations including polynomials]**, the modulus of a linear function, **[** $y=\frac{a}{x}$ **and** $y=\frac{a}{x^2}$ **(including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations]** **[Understand and use proportional relationships and their graphs]**	Can we sketch the graph $y=x3-x^2-x+1$?		
B8	Understand and use composite functions; inverse functions and their graphs			
B9	**[Understand the effect of simple transformations on the graph of** $y=\mathrm{f}(x)$ **including sketching associated graphs:** $y=a\mathrm{f}(x)$, $y=\mathrm{f}(x)+a$, $y=\mathrm{f}(x+a)$, $y=\mathrm{f}(ax)$**]**, and combinations of these transformations			
B10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear)			
B11	Use of functions in modelling, including consideration of limitations and refinements of the models			

C	**Coordinate geometry in the (x,y) plane**			
	Content			
C1	**[Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular]** **[Be able to use straight line models in a variety of contexts]**	Can we find a rectangle from two vertices and a diagonal?	Can we find the point midway between an intersection and a line?	Given a right angle and the equations of two sides, can we find the third?
C2	**[Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x + a)^2 + (y + b)^2 = r^2$; completing the square to find the centre and radius of a circle; use of the following properties:** **the angle in a semicircle is a right angle** **the perpendicular from the centre to a chord bisects the chord** **the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point**	Can we show that these two circles touch?	Can we show that these four points lie on a circle?	
C3	Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms	What's the locus of the intersection of these variable lines?		
C4	Use parametric equations in modelling in a variety of contexts			
D	**Sequences and series**			
	Content			
D1	**[Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and ${}_nC_r$; link to binomial probabilities]** Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\| < 1$. (proof not required)	Can we use a binomial expansion to evaluate $(19\ 3/4)^6$	Can we estimate a difference of eighth powers?	Which power of xx has the greatest coefficient?
D2	Work with sequences including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$; increasing sequences; decreasing sequences; periodic sequences	What is this multiple of 13's final digit?		
D3	Understand and use sigma notation for sums of series			
D4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms	When does the sum of this series equal 6060?		

D5	Understand and work with geometric sequences and series including the formulae for the *n* th term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation	Can we sum the first 2n terms of 1,1,2,1/2,4,1/4,8,1/8,..?		
D6	Use sequences and series in modelling			

E Trigonometry

	Content			
E1	**[Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form** $\frac{1}{2}ab\sin C$ **]** Work with radian measure, including use for arc length and area of sector	Are any of these four trig expressions equal to each other? Can we find the area between three touching circles?	How are these three angles connected?	How much of this log is below the surface?
E2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta,\ \cos\theta \approx 1-\frac{\theta^2}{2},\ \tan\theta \approx \theta$ where θ is in radians			
E3	**[Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity]** Know and use exact values of sin and cos for $0,\ \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3},\ \frac{\pi}{2},\ \pi$ and multiples thereof, and exact values of tan for $0,\ \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3},\ \pi$ and multiples thereof	What's the result when we reflect y=sinxy=sinx twice	An exact value for sin15	Which of these log and trig expressions is the largest?
E4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains			
E5	**[Understand and use** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **]** **[Understand and use** $\sin^2\theta + \cos^2\theta = 1$**]**; $\sec^2\theta = 1+\tan^2\theta$ and $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	How many solutions does 2cos2x+5sinx=4 have?	Can we prove this trig identity to solve this equation?	
E6	Understand and use double angle formulae; use of formulae for $\sin(A\pm B),\ \cos(A\pm B)$ and $\tan(A\pm B)$; understand geometrical proofs of these formulae Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta\pm\alpha)$ or $r\sin(\theta\pm\alpha)$	Can we write sinθ and cosθ in terms of tan(θ/2)?	For which 0<θ<500 is 3cosθ+4sinθ >2?	Can we find the turning points on the curve y=sinx+cosx?
E7	**[Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle]**	Can we solve the equation 3cosx+1=2sinx?	How many solutions does cos(sinx)=0.5 have?	

E8	Construct proofs involving trigonometric functions and identities			
E9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces	Can we find the distance of the tree from the road?		

F	Exponentials and logarithms			
	Content			
F1	[Know and use the function a^x and its graph, where a is positive] [Know and use the function e^x and its graph]			
F2	[Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications]			
F3	[Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$] [Know and use the function $\ln x$ and its graph] [Know and use $\ln x$ as the inverse function of e^x]	Can we find these log expressions in terms of $\log_a x$?	Given this log equation, when can aa be largest?	
F4	[Understand and use the laws of logarithms: $\log_a x + \log_a y = \log_a(xy)$; $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$; $k\log_a x = \log_a x^k$ (including, for example, $k = -1$ and $k = -\frac{1}{2}$)]	Can we use the laws of logs to find x and y?	Can we evaluate these log expressions without using a calculator?	
F5	[Solve equations of the form $a^x = b$]	How many real roots does 8x+4=4x+2x+2have?	When does 9x−3x+1=k have one or more real solutions?	Can we solve these simultaneous exponential equations?
F6	[Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y]			
F7	[Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models]			

G	Differentiation			
	Content			
G1	**[Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y);the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x]** and for $\sin x$ and $\cos x$ **[Understand and use the second derivative as the rate of change of gradient]**; connection to convex and concave sections of curves and points of inflection	Can we maximise the volume of this metal tin?		
G2	**[Differentiate x^n, for rational values of n, and related constant multiples, sums and differences]** Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples Understand and use the derivative of $\ln x$	What happens if we expand, then differentiate?	What is the gradient of this curve at its x-intercepts?	Can we find the turning points on the curve y=sinx+cosx?
G3	**[Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points]**, points of inflection **[Identify where functions are increasing or decreasing]**	Is the gradient function increasing or decreasing on this curve?	Can we find the locus of a midpoint created by a parabola?	What's the most economical speed for this ship's journey?
		Can we show the function (2x+1)/(x2−1)(2x+1)/(x2−1) can take all real values?		
G4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions	Can we show x>ln(1+x2) for all x>0?	Can we track this constantly growing patch of fluid?	
G5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only	Can we find this area by integrating with respect to y?	Can we find the equation of the normal to the curve when t=2?	Can we find where the normal is parallel to y=x?
G6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand)			

H	**Integration**			
	Content			
H1	**[Know and use the Fundamental Theorem of Calculus]**	Can we find a curve from its gradient function?		
H2	**[Integrate x^n (excluding n = -1), and related sums, differences and constant multiples]** Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples	Can we solve these simultaneous equations with integrals?	Can we find the area between sinx and sin2x?	Where do the curves y=sin2x and y=sinx cross?
H3	**[Evaluate definite integrals; use a definite integral to find the area under a curve]** and the area between two curves	Can we explain why this integral is zero?	Can we find the area enclosed by this quadratic and the xx-axis?	Can we find the area inside a parabola, a tangent, and the x-axis?
		Can we find the area between a parabola and a line?		
H4	Understand and use integration as the limit of a sum			
H5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae)	Can we use integration by substitution on these integrals?		
H6	Integrate using partial fractions that are linear in the denominator			
H7	Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor)			
H8	Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics			

I	**Numerical methods**			
	Content			
I1	Locate roots of $f(x)=0$ by considering changes of sign of $f(x)$ in an interval of x on which $f(x)$ is sufficiently well-behaved Understand how change of sign methods can fail			
I2	Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1}=g(x_n)$ Understand how such methods can fail			
I3	Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between			
I4	Use numerical methods to solve problems in context			
J	**Vectors**			
	Content			
J1	**[Use vectors in two dimensions]** and in three dimensions	When are these vectors parallel/perpendicular/the same length?		
J2	**[Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form]**			
J3	**[Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations]**			
J4	**[Understand and use position vectors; calculate the distance between two points represented by position vectors]**			
J5	**[Use vectors to solve problems in pure mathematics and in context, including forces]** and kinematics	What is the position vector of DD if ABCDABCD is a parallelogram?		

	For sections K to O students must demonstrate the ability to use calculator technology to compute summary statistics and access probabilities from standard statistical distributions.			

K	**Statistical sampling**			
	Content			
K1	**[Understand and use the terms 'population' and 'sample']** **[Use samples to make informal inferences about the population]** **[Understand and use sampling techniques, including simple random sampling and opportunity sampling]** **[Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population]**			
L	**Data presentation and interpretation**			
	Content			
L1	**[Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency]** **[Connect to probability distributions]**			
L2	**[Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded)]** **[Understand informal interpretation of correlation] [Understand that correlation does not imply causation]**			
L3	**[Interpret measures of central tendency and variation, extending to standard deviation]** **[Be able to calculate standard deviation, including from summary statistics]**			
L4	**[Recognise and interpret possible outliers in data sets and statistical diagrams]** **[Select or critique data presentation techniques in the context of a statistical problem]** **[Be able to clean data, including dealing with missing data, errors and outliers]**			
M	**Probability**			
	Content			
M1	**[Understand and use mutually exclusive and independent events when calculating probabilities]** **[Link to discrete and continuous distributions]**			

M2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables Understand and use the conditional probability formula $\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$			
M3	Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions			

N	**Statistical distributions**			
	Content			
N1	**[Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution]**			
N2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution Link to histograms, mean, standard deviation, points of inflection and the binomial distribution			
N3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate			
O	**Statistical hypothesis testing**			
	Content			
O1	**[Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p* -value]**; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded)			
O2	**[Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context]** **[Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis]**			
O3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context			

P	**Quantities and units in mechanics**			
	Content			
P1	**[Understand and use fundamental quantities and units in the S.I. system: length, time, mass] [Understand and use derived quantities and units: velocity, acceleration, force, weight],** moment			
Q	**Kinematics**			
	Content			
Q1	**[Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration]**			
Q2	**[Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph]**	At what time would these two trains meet?	Can we calculate the total time taken for this car journey?	What's the average speed of the second car
Q3	**[Understand, use and derive the formulae for constant acceleration for motion in a straight line]**; extend to 2 dimensions using vectors			
Q4	**[Use calculus in kinematics for motion in a straight line:** $v = \frac{dr}{dt}$, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$, $r = \int v\,dt$, $v = \int a\,dt$ **]**; extend to 2 dimensions using vectors			
Q5	Model motion under gravity in a vertical plane using vectors; projectiles			

R	**Forces and Newton's laws**			
	Content			
R1	**[Understand the concept of a force; understand and use Newton's first law]**			
R2	**[Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors)]**; extend to situations where forces need to be resolved (restricted to 2 dimensions)	Given the position of a mass, can we find its velocity and acceleration?		
R3	**[Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g* , and its value in S.I. units to varying degrees of accuracy]** **[(The inverse square law for gravitation is not required and *g* may be assumed to be constant, but students should be aware that *g* is not a universal constant but depends on location)]**			
R4	**[Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles]**; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces			
R5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane			
R6	Understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics			
S	**Moments**			
	Content			
S1	Understand and use moments in simple static contexts			