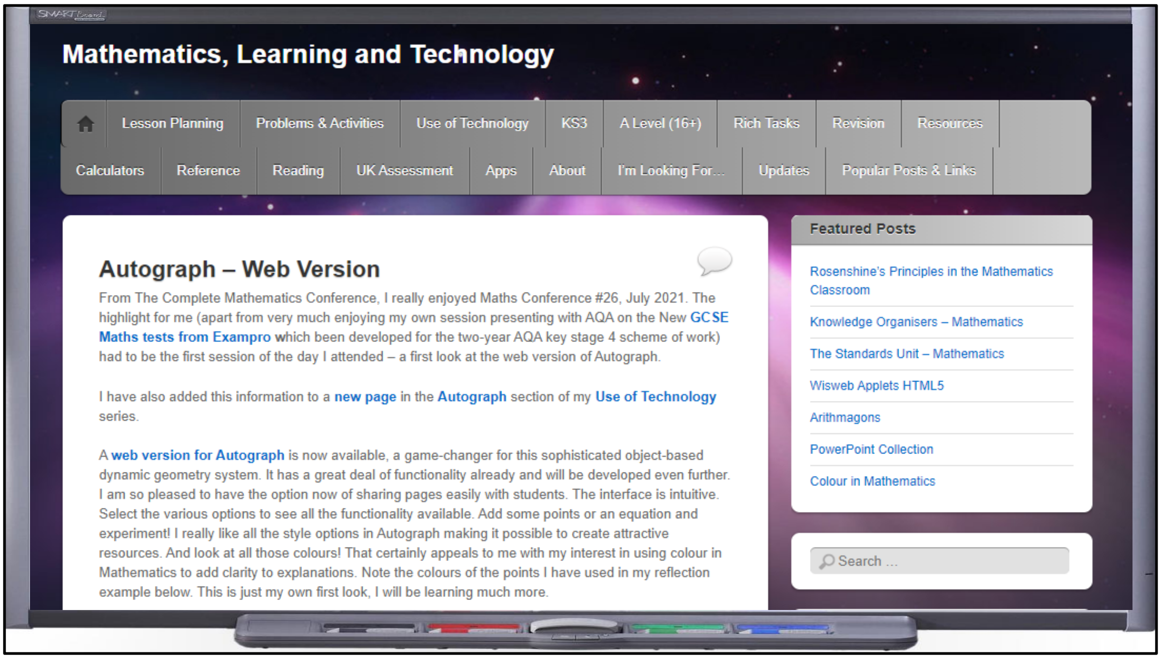
Colour in Mathematics

The use of **colour** to add clarity

Colleen Young
colleenyoung.org
Twitter: @ColleenYoung



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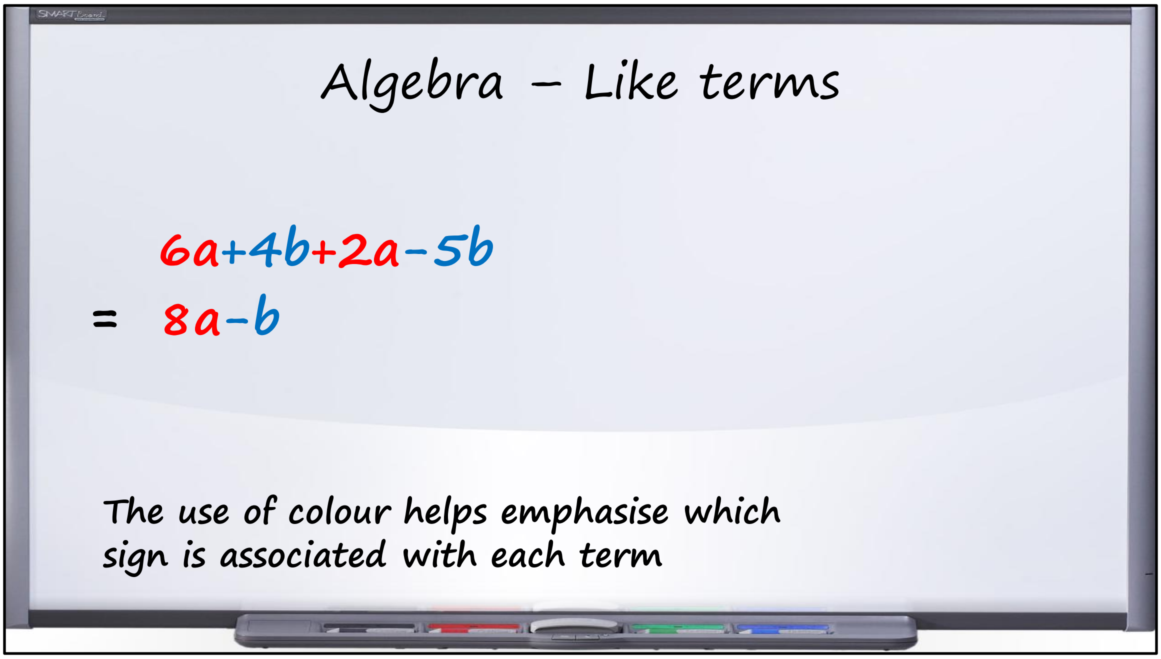
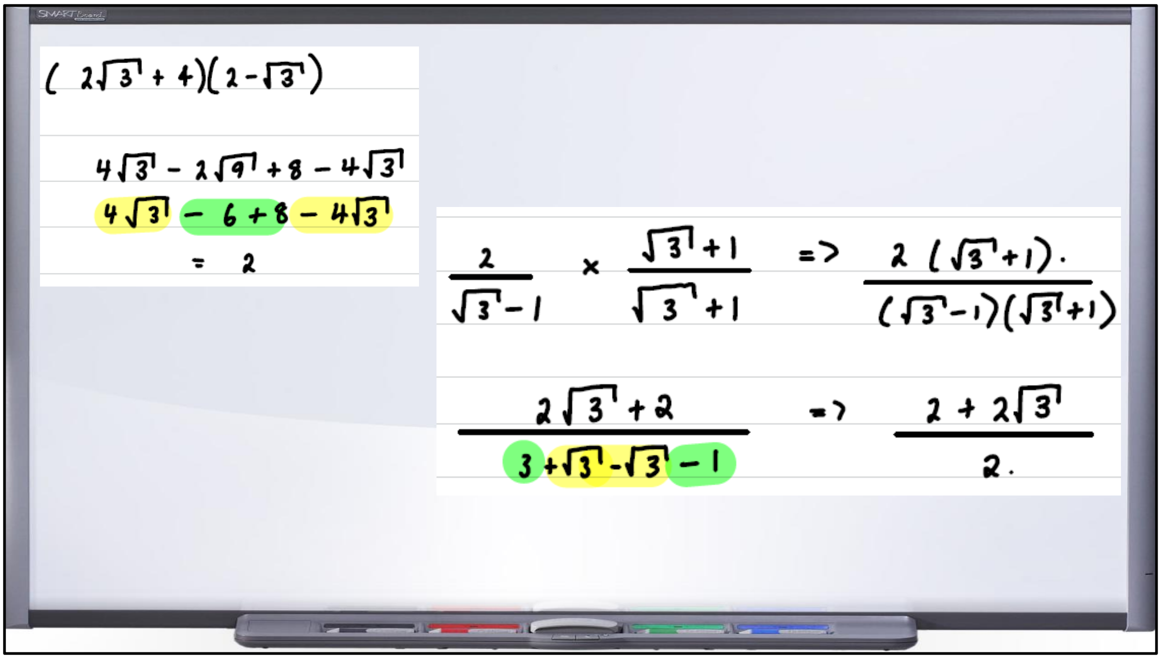
Note the featured posts on the right, Colour in Mathematics is available now here.

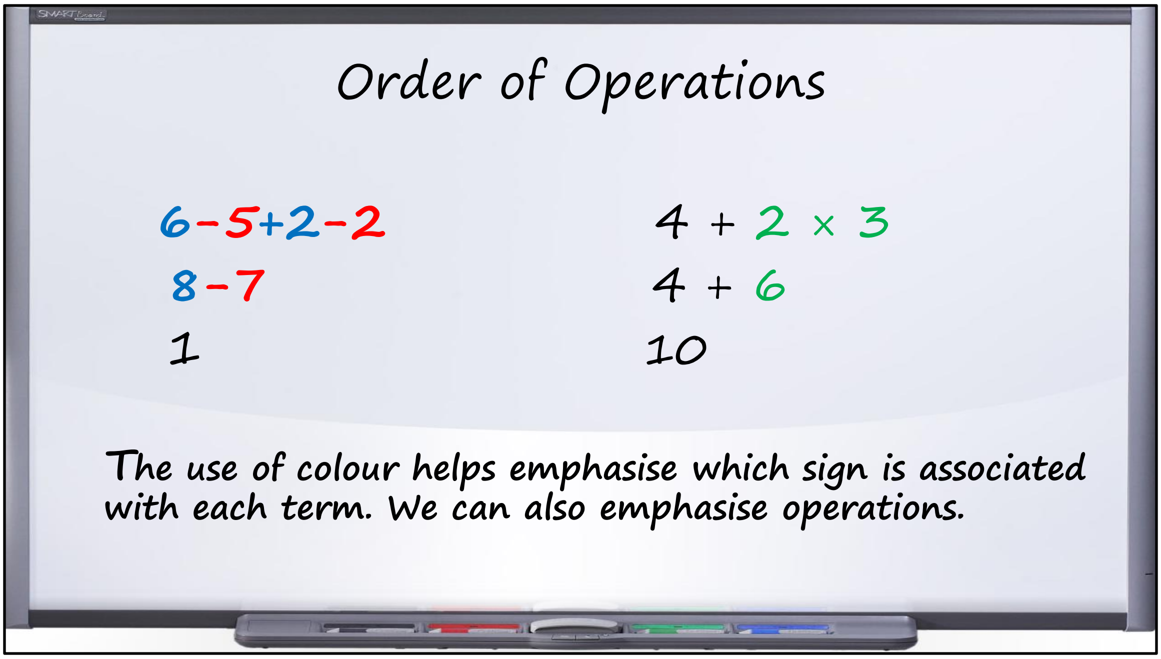


This is where it all began, as a young teacher I found so many students do not appreciate which sign is associated with which term. Colour emphasises this.

Teaching this I have seen some students, particularly the younger students learning simplification of terms for the first time actually doing this themselves and identifying like terms in colour. Deliberate use of colour helps them identify like terms.

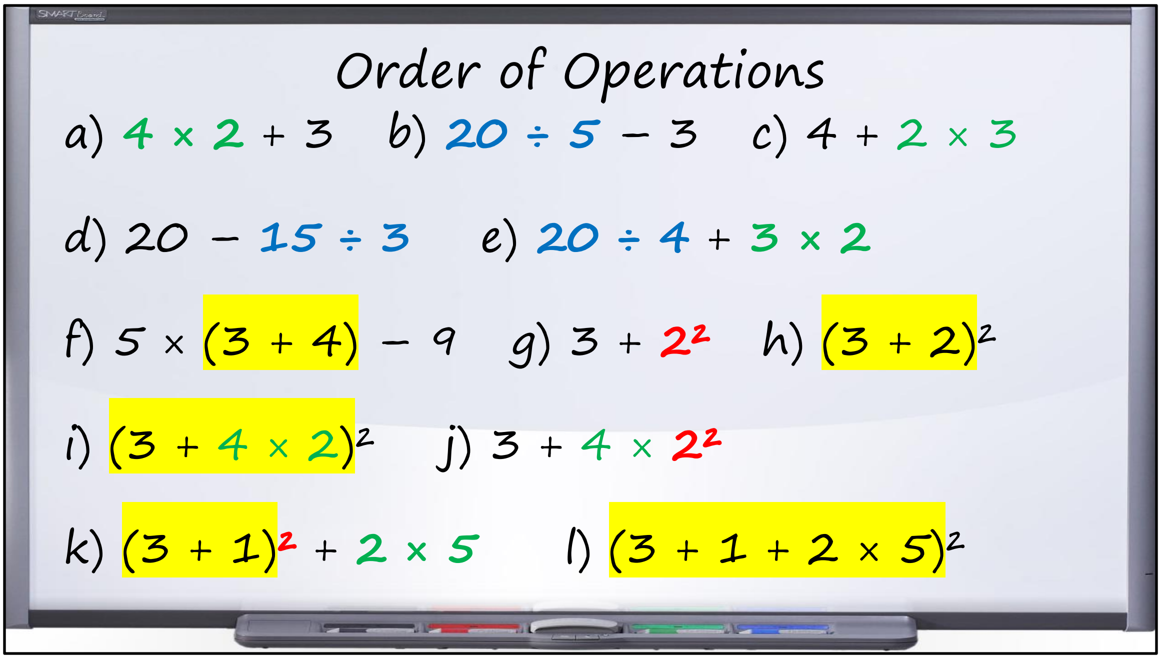


As a contrast to beginning algebra, this is the work of one of my distance learning students, who I noted on his recent A level assignment was using colour well to help his own thinking.



Similarly with numerical examples, it is important for students to appreciate which sign is associated with each number, which we can see in my example on the left.
I have often pointed out to them that we would indeed get the same answer if we did for example
6-5+2-2
6+2-5-2

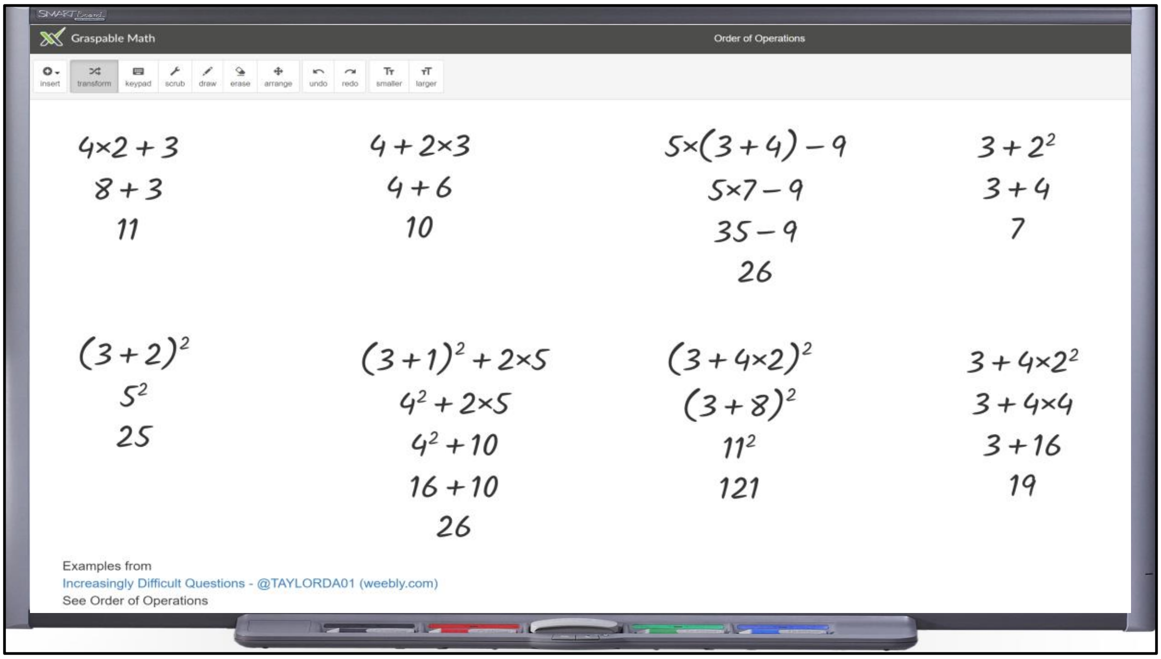
With order of operations as on the right here, I have also used colour when first going through examples together, or perhaps revising later to emphasise what should be done first.



Some further examples I thought about – how colour could be used as a reminder of the order of operations. Trying to pick out what to concentrate on first here. Highlighting can be a useful technique also.
I might show this sort of example when introducing topics, also when going through exercises.

The red colour here helps student to recall that indices need careful treatment.

Now because a theme of mine is how technology can enhance learning – I can't resist a nod to Graspable Math…

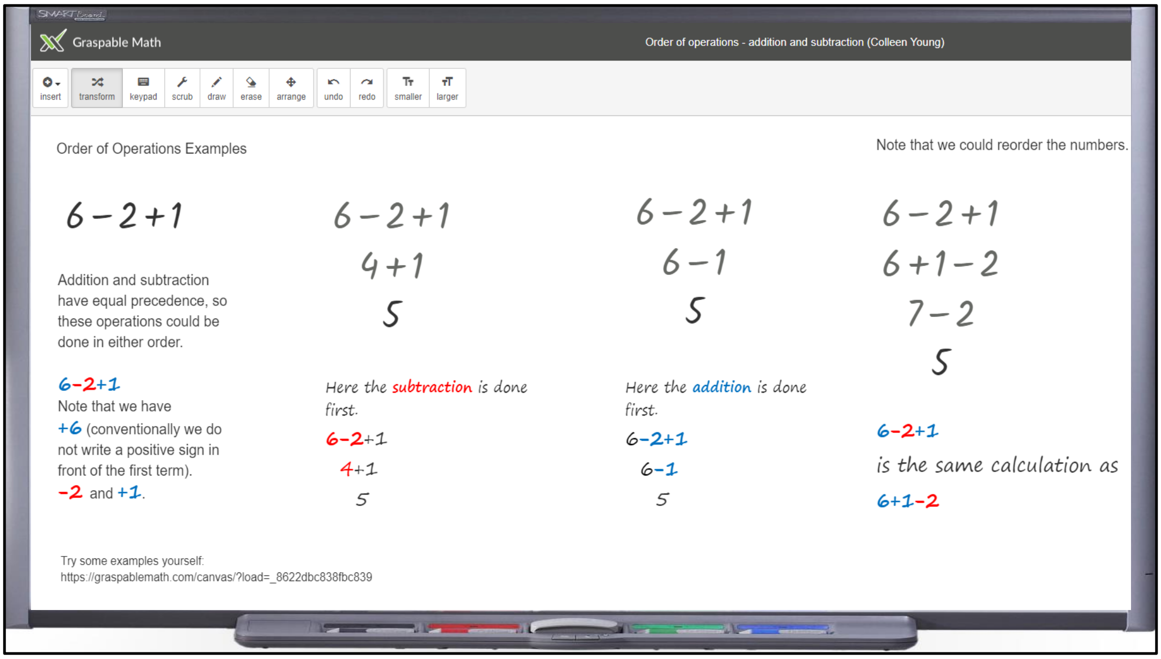


Using a graspable math canvas, you are only allowed to evaluate expressions using the correct order.

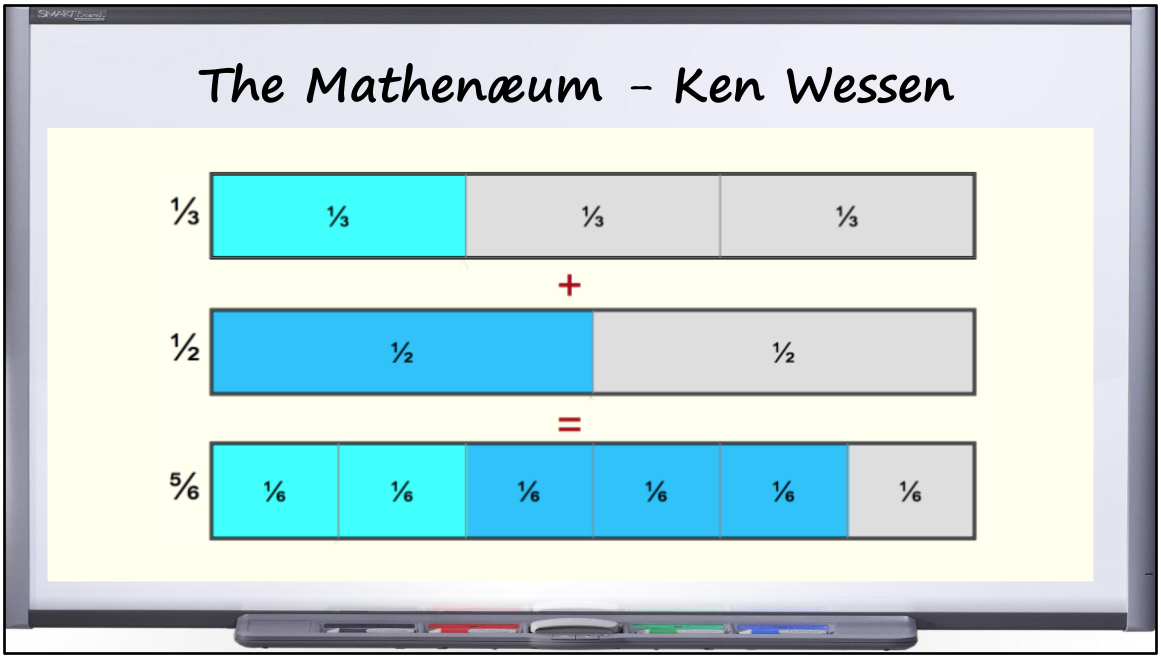
If you have not used Graspable Math before – touching the operator will perform that operation.

In that second example, 4+2x3, if you touch the + first the numbers will just jump about but no operation will be performed. Touch the x though and 2x3 will be calculated.

A great way to demonstrate in class as you can show each line of working.

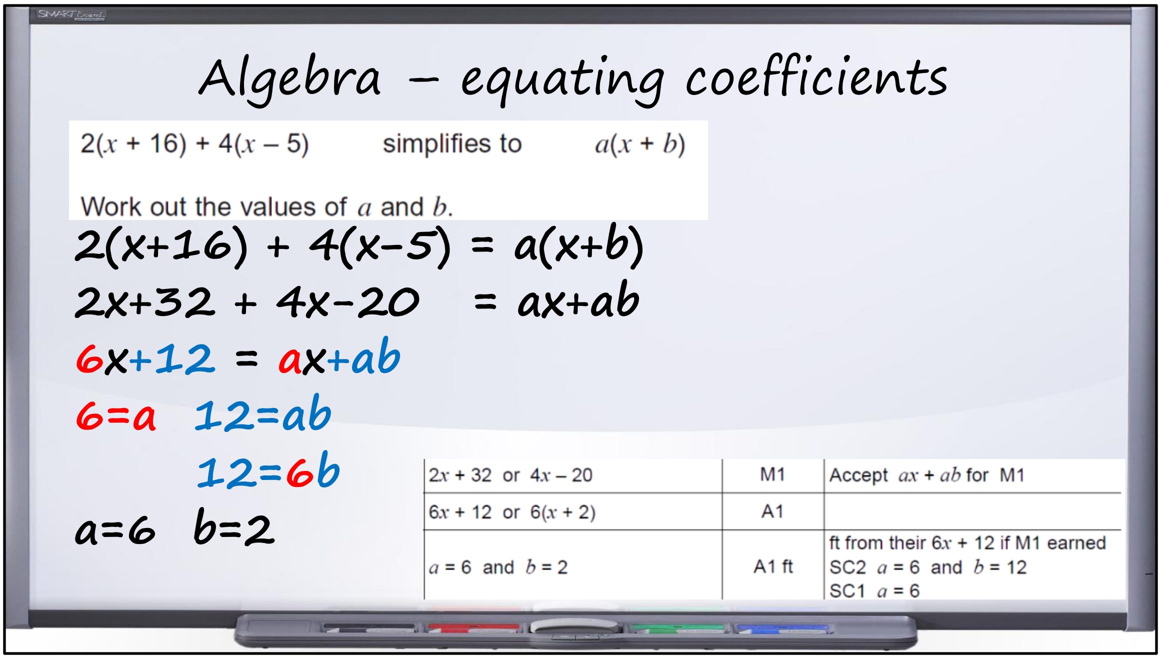


It is possible to add images and text to a Graspable Math canvas, I created this for my students to see some more examples and then they could try some further examples themselves – note the link to another canvas.



Further examples now – a mixture of my own examples and examples of resources where colour is used very well to add clarity.

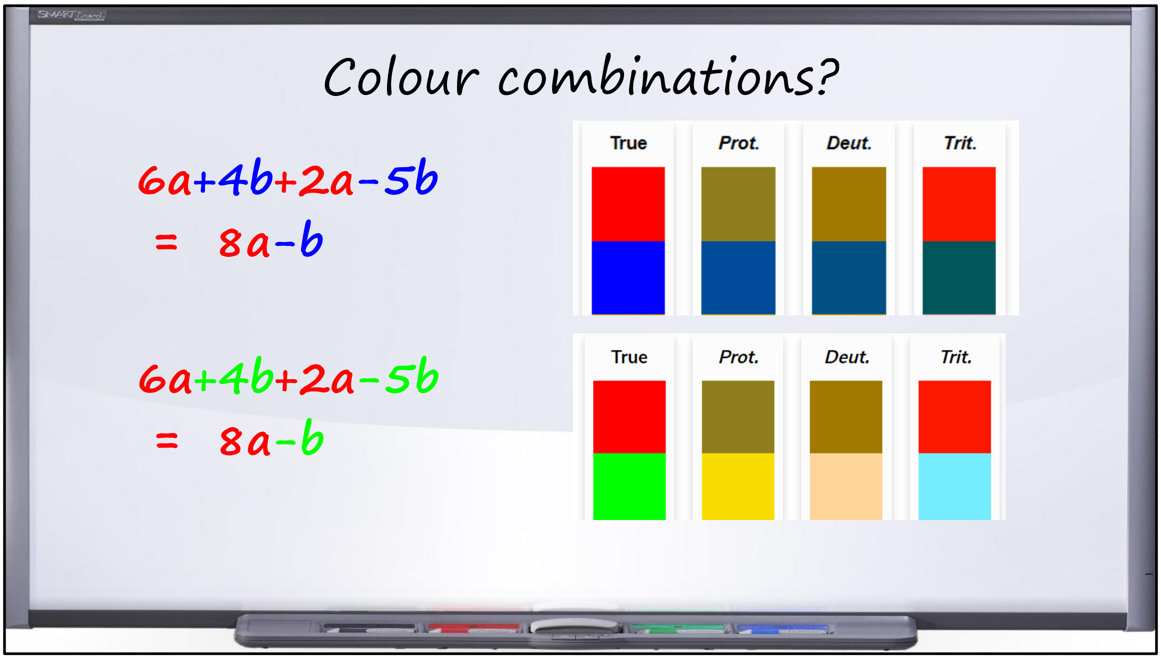
On Ken Wessen's The Mathenaeum his Fractions, Decimals and percentages includes diagrams like this. I like the simplicity of this, the colour helping the explanation.



More Algebra, this is a GCSE question (AQA).

Certainly colour is helpful when equating coefficients.
Also relevant for A level.

I have been asked before about colour blindness …

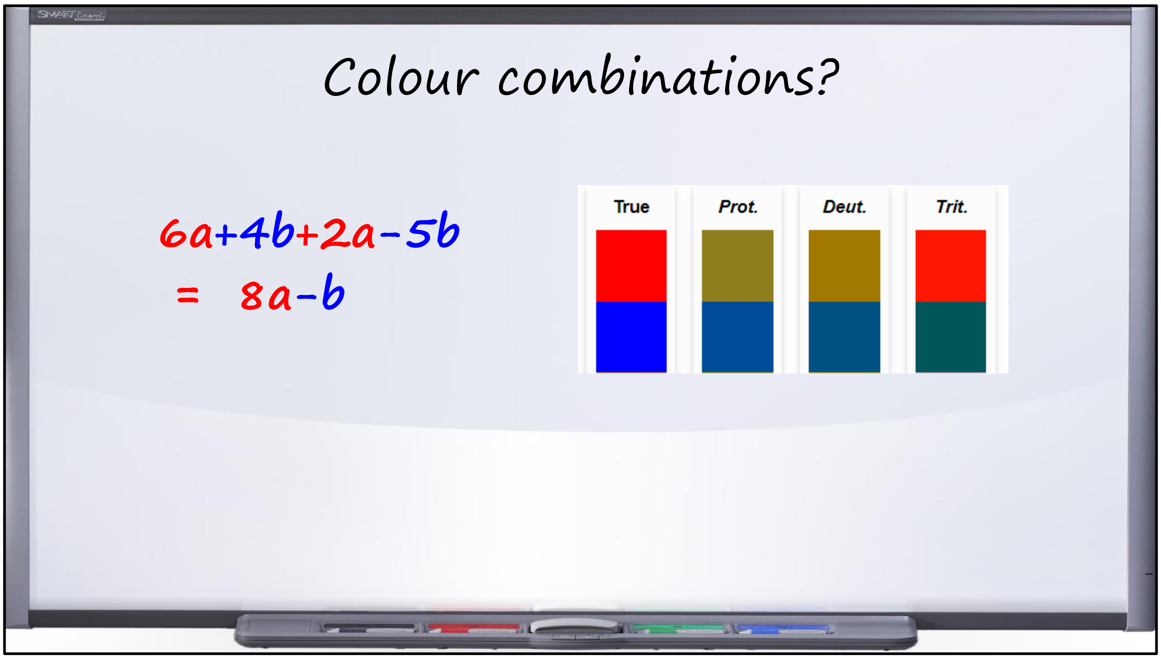


According to Color Blind Awareness, color blindness affects 1 in 12 men (8%) and 1 in 200 women (0.5%). There are an estimated 300 million colour blind people worldwide.

To quote from that site…
"And that doesn't mean banishing colour. More than 99% of color blind people can, in fact, see colour—just not in the same way as someone who isn't impacted by colour blindness."

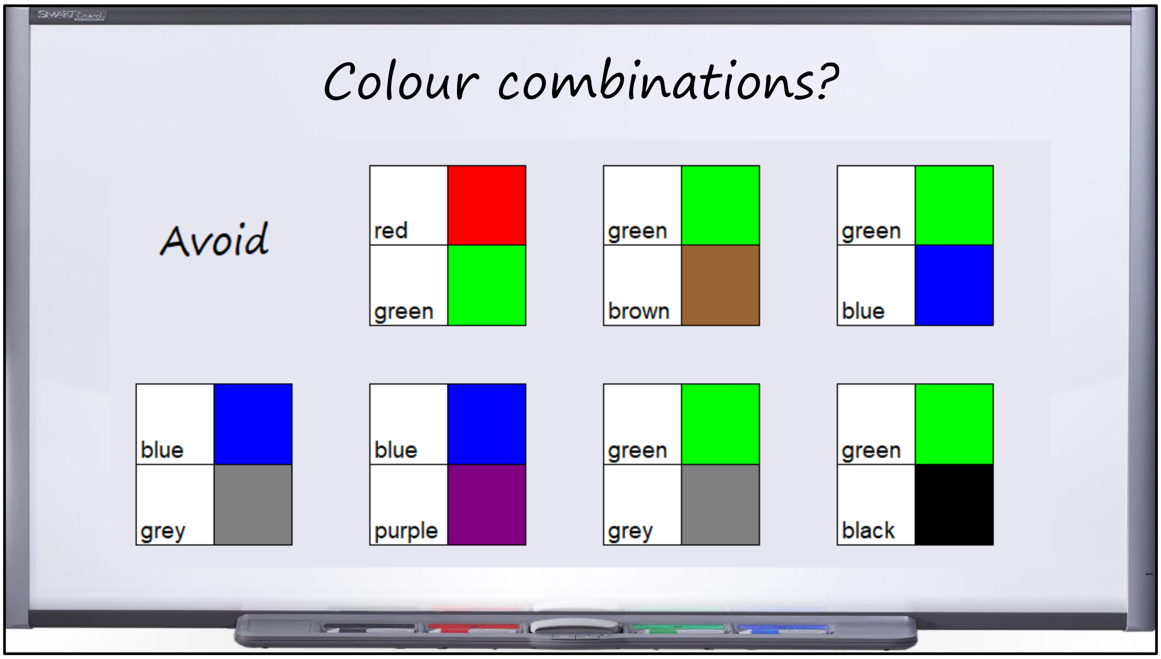
I used a website to simulate what my chosen colour combination looks like to viewers who are colourblind. The colours in the leftmost column are the "true" colours; those displayed in the remaining three columns show the way that a person with different types of colour blindness, oprotanopia, deuteranopia, or tritanopia would see them.

So my red/blue at the top, it seems as though whilst colour blind people won't see red and blue as I see it, but they will see two contrasting colours. Red/green is a common problem, it seems as though there is still a contrast, but I don't think those combinations are very clear. That green is too bright and does not show up well for anyone in my view! (red 0 blue 0 and green 255)



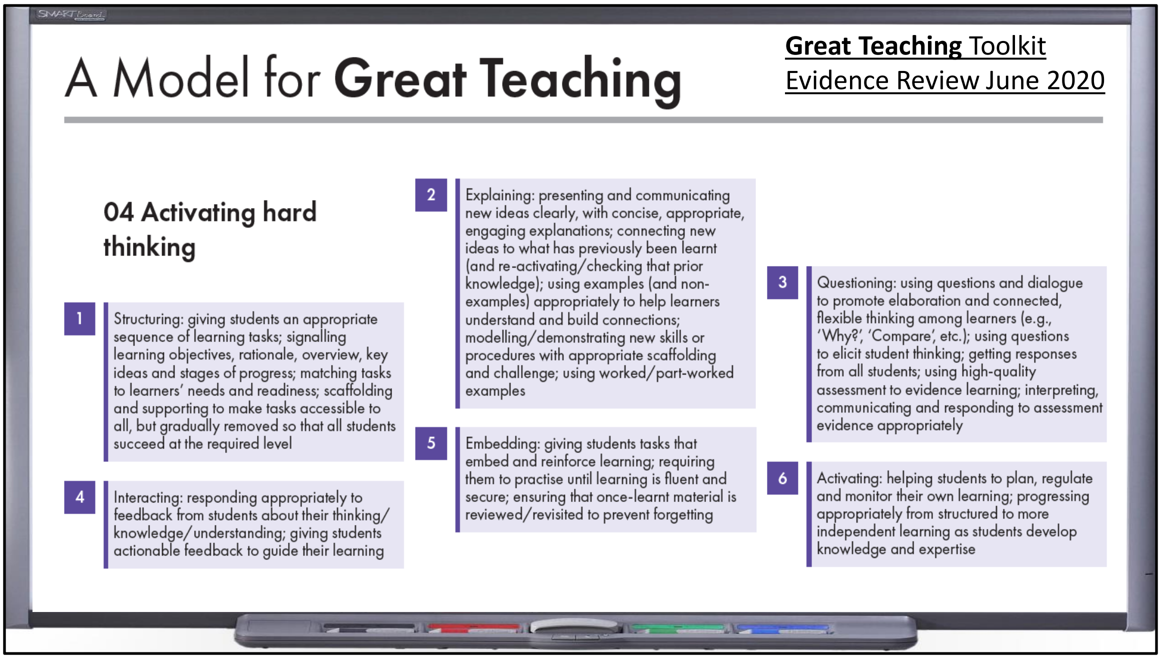
Red / blue all combinations seem dark enough and a contrast.

(Find out more about any individuals in your class who have visual problems; experiment with combinations which work for them.)



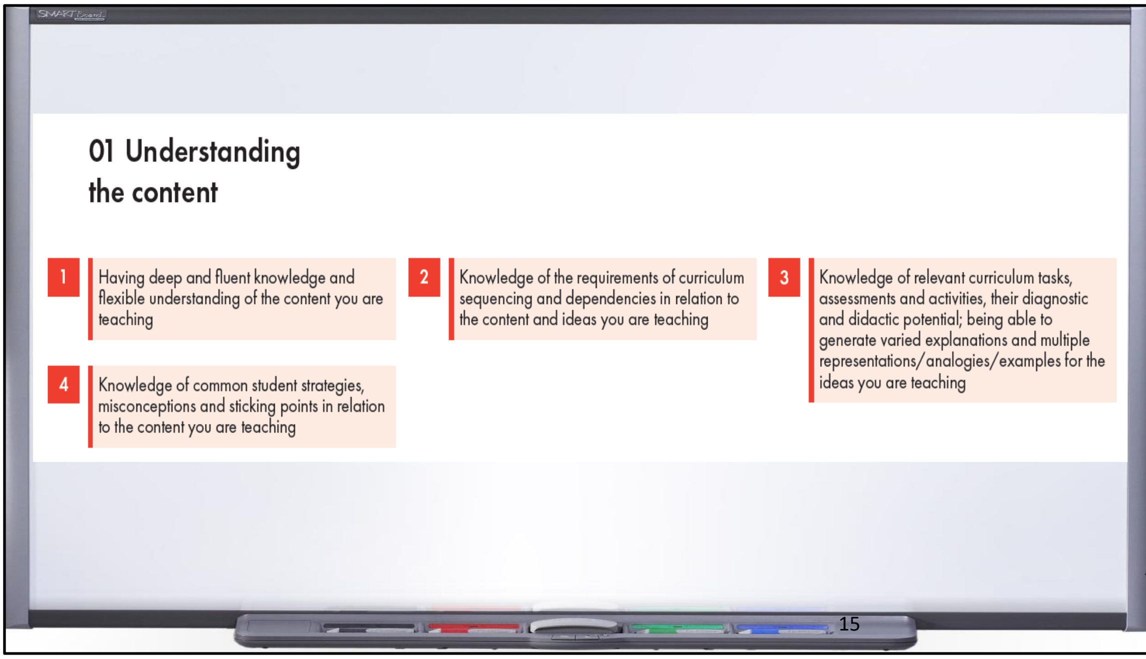
Good to be aware of **Color combinations to avoid for people with colour blindness include:**

•Red & green
•Green & brown
•Green & blue
•Blue & grey
•Blue & purple
•Green & grey
•Green & black



At this point I would like to mention both the Great Teaching Toolkit Evidence Review and also Rosenshine's Principles of Instruction.
In the Great Teaching Toolkit Evidence Review Professor Rob Coe and his team at Evidence Based Education have drawn from a wide range of research, examined hundreds of pieces of research on the links between teacher performance and student outcomes.
What really makes a difference? What should teachers focus on?
A model is presented in the paper with 4 dimensions, with 17 elements within those dimensions.
An 'element' is defined as something that may be worth investing time and effort to work on to build a specific competency, skill or knowledge, or to enhance the learning environment.

Thinking about using colour for explanations I believe helps address element 2, Explaining: presenting and communicating new ideas clearly, with concise, appropriate engaging explanations.



For me, dimensions 1 understanding the content and 4 activating hard thinking go hand in hand, we have to have that deep subject knowledge in order to explain it to our students.

Understanding the content well helps us in understanding the sort of explanations which will help our students. For example I realised as a young teacher that many students do not appreciate the fact that the sign in front of a term belongs to that term. Understanding those misconceptions helps us in planning our explanations.

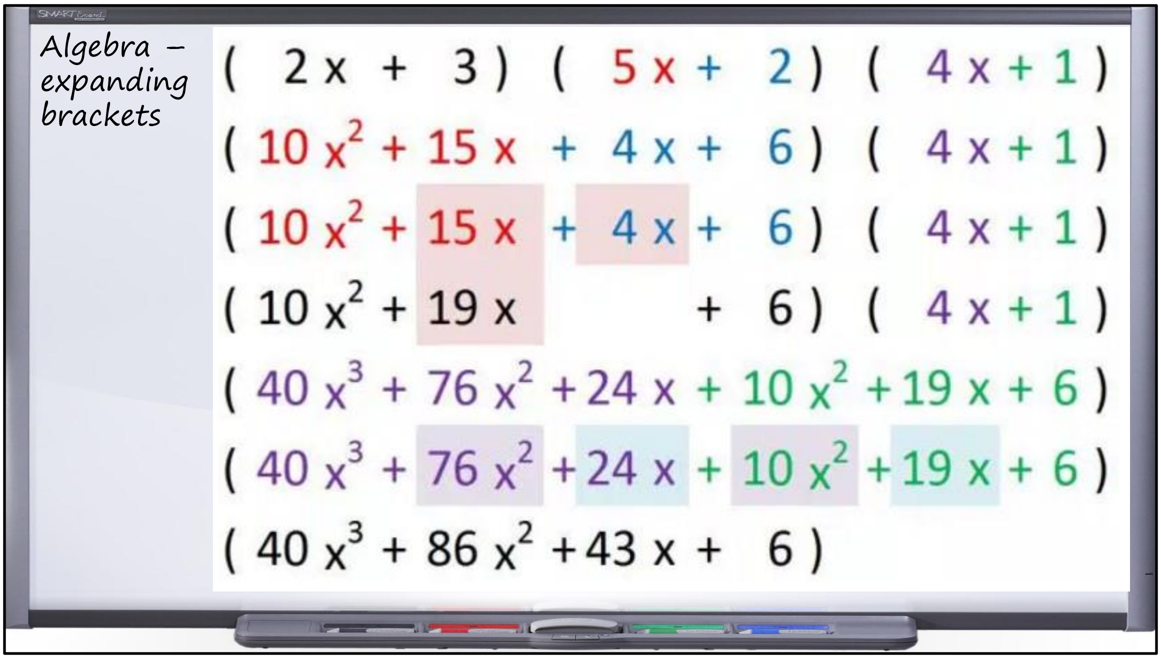
There are many links with Rosenshine here. In fact the review drew on much research including Rosenshine.

17 Principles of Effective Instruction	**9 Provide models of worked-out problems.**
1 Begin a lesson with a short review of previous learning.	10 Ask students to explain what they have learned.
2 Present new material in small steps with student practice after each step.	11 Check the responses of all students.
3 Limit the amount of material students receive at one time.	12 Provide systematic feedback and corrections.
4 Give clear and detailed instructions and explanations.	**13 Use more time to provide explanations.**
5 Ask a large number of questions and check for understanding.	**14 Provide many examples.**
6 Provide a high level of active practice for all students.	15 Reteach material when necessary.
7 Guide students as they begin to practice.	**16 Prepare students for independent practice.**
8 Think aloud and model steps.	17 Monitor students when they begin independent practice.

If you have seen Rosenshine's Principles you are probably now thinking – there's 10 principles..!

There are, but note in his paper, Principles of Instruction: research-based strategies that all teachers should know (2012) American Educator Barak Rosenshine presents this list of 17 principles emerging from his research; I find this provides useful detail.

Now thinking about using colour in explanations is useful when thinking about several principles here, obviously much more could be said about each but I believe good use of colour can certainly help us give clear and detailed explanations.

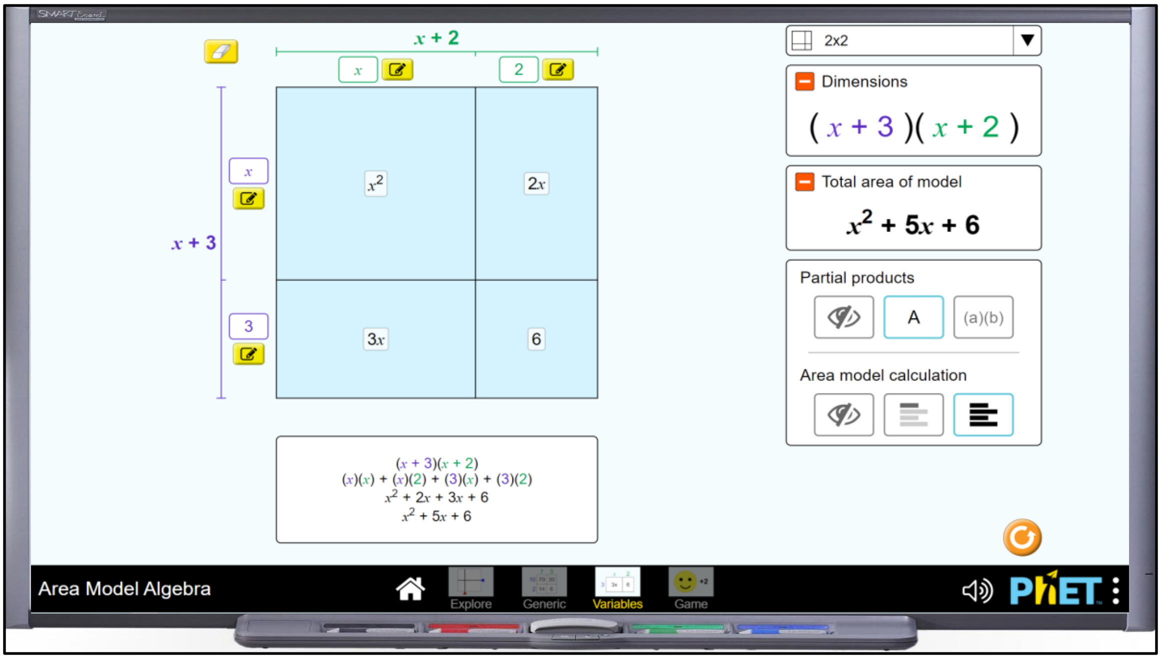


Appearing on the GCSE specification we have multiplication of 2 ***or more*** binomials.
Working through this example with a class I used colour to keep track of where we were.

Multiplying the first two brackets first we'll multiply both terms in the first bracket by 5x and then by 2 …..and so on.

Excel is useful for preparing an example like this as we can easily format the text or the cell colour.

Obviously this could be done with just two brackets also.



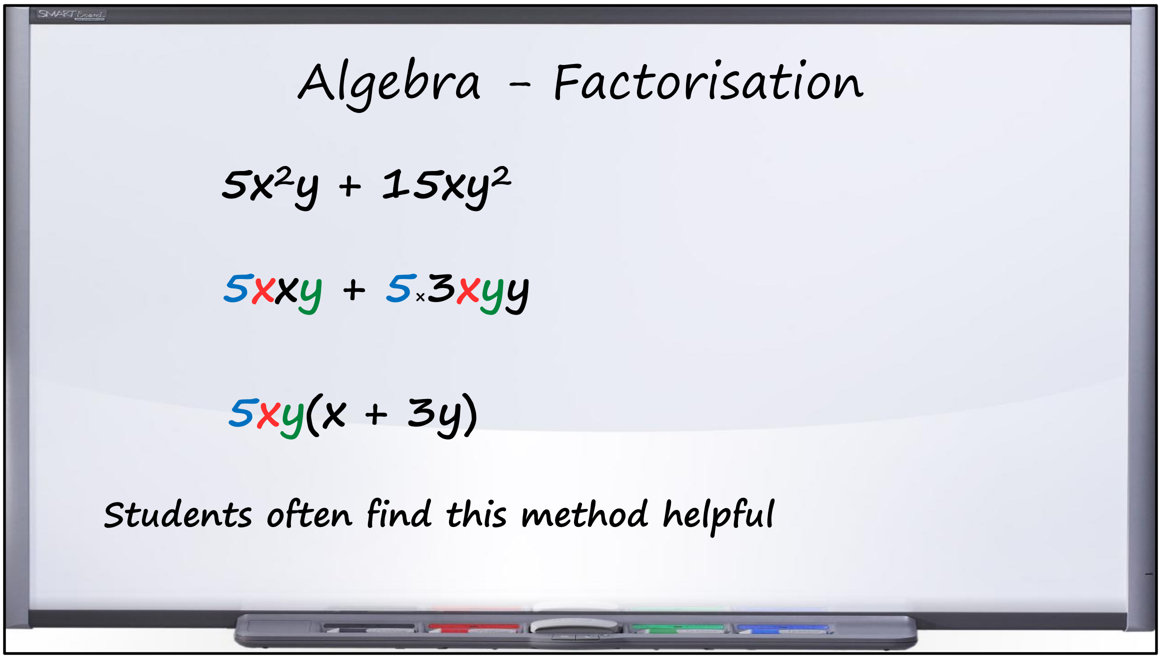
A further example – Area Model algebra. PhET SIMS has used colour clearly.

Alternative use of colour here with x+3 in one colour and x+2 in a second colour.

These are very attractively presented. Note the choice of what is visible. You can choose to display the working shown for example.

Working clearly shows taking a term from each bracket and of course is also illustrated by the diagram.

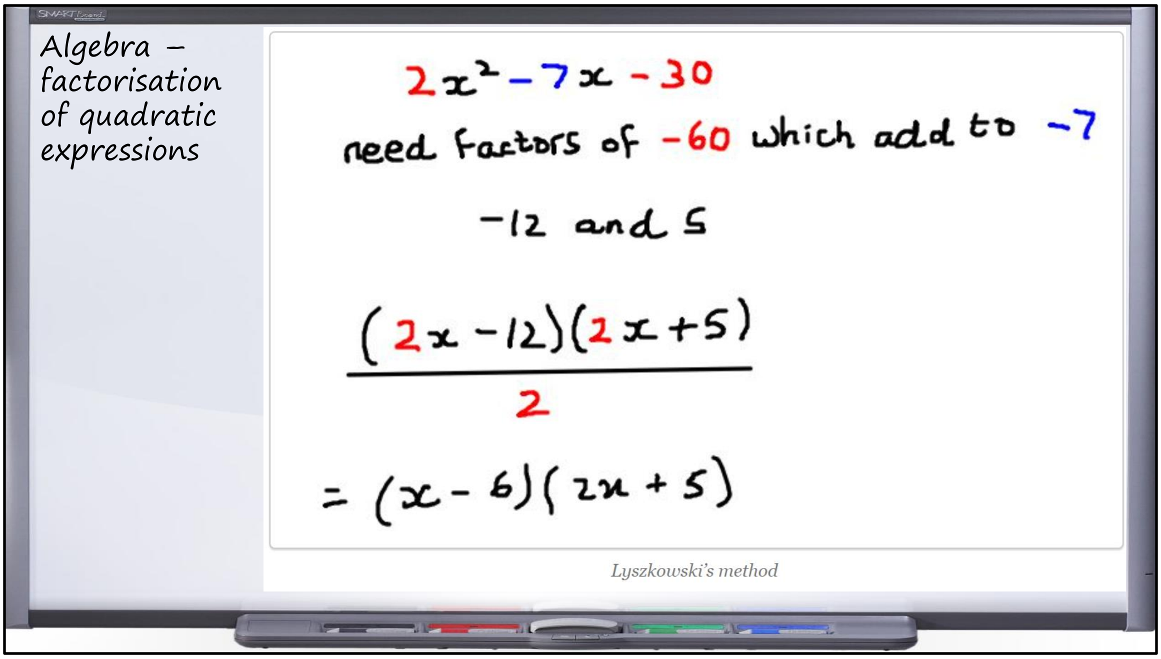
Ideal you can choose the dimensions yourself.
Students could try the example themselves then use this area model to check.



Now this became known as “Mrs Young’s factorisation method” which many students who had struggled with factorisation really like.

Write it all out in long form then look for common factors. Again I have seen this in books, I think it’s when you see it years later you know that it worked for them. Many don’t need it of course, but if it helps some… that’s good.

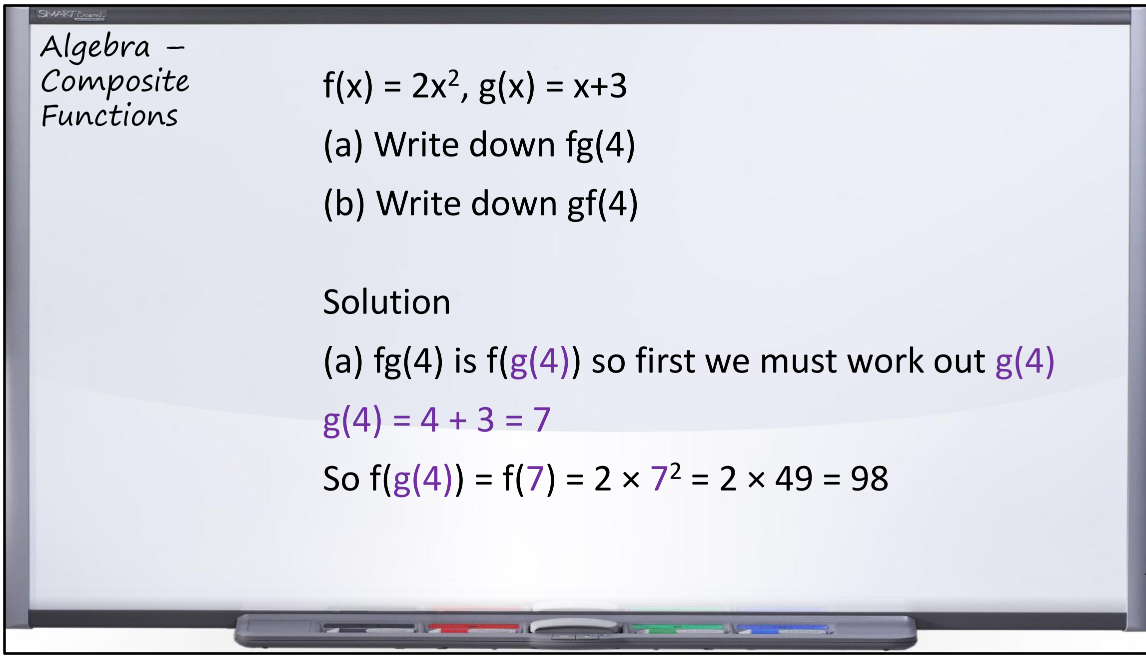
(Test red/green! Lighter red and darker green)



On factorisation – I must show a favourite method to use when the coefficient of x squared is not 1.
I must emphasise I would never say that one method is ***the best***.
If the coefficient of x squared and the constant are prime then just get on and do it by inspection.

Some students get in a mess when splitting the middle term, I have found Lyszkowski's method appeals to them.

Underground Maths task and Colin Foster. (See blog post)



With composite functions I have found colour really helps.
I wrote this for a distance learning student of mine recently, she was struggling with the explanation she was reading in a text.

She said that that this helped where the more wordy explanation had puzzled her.

Algebra – Composite Functions

$f(x) = 2x^2$, $g(x) = x+3$

(a) Write down $fg(4)$

(b) Write down $gf(4)$

Solution

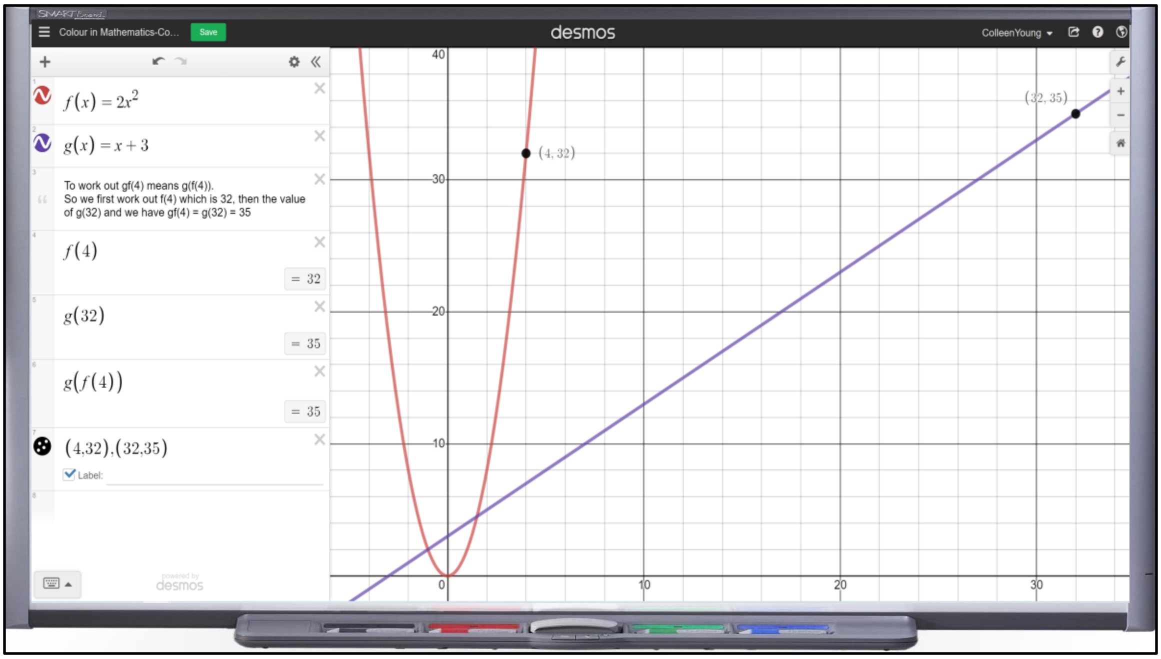
(b) $gf(4)$ is $g(f(4))$ so first we must work out $f(4)$

$f(4) = 2 \times 4^2 = 2 \times 16 = 32$

So $g(f(4)) = g(32) = 32 + 3 = 35$

Note that we can check values of functions on Desmos …

Note…check on Desmos



…and of course we can check functions on Desmos too.

Note the brackets needed for eg g(f(x))

Desmos useful for complete worked examples .
As notes and links can be added.

We can change the colours in Desmos – here I have been consistent with the choice of colours for each function for the series of slides.

Obviously can use Classroom activities also.

Algebra – Composite Functions

$f(x) = 2x^2$, $g(x) = x+3$

(c) Write down $fg(x)$

(d) Write down $gf(x)$

Solution

Looking at the algebra now to find $f(g(x))$, we need $f(g(x))$.

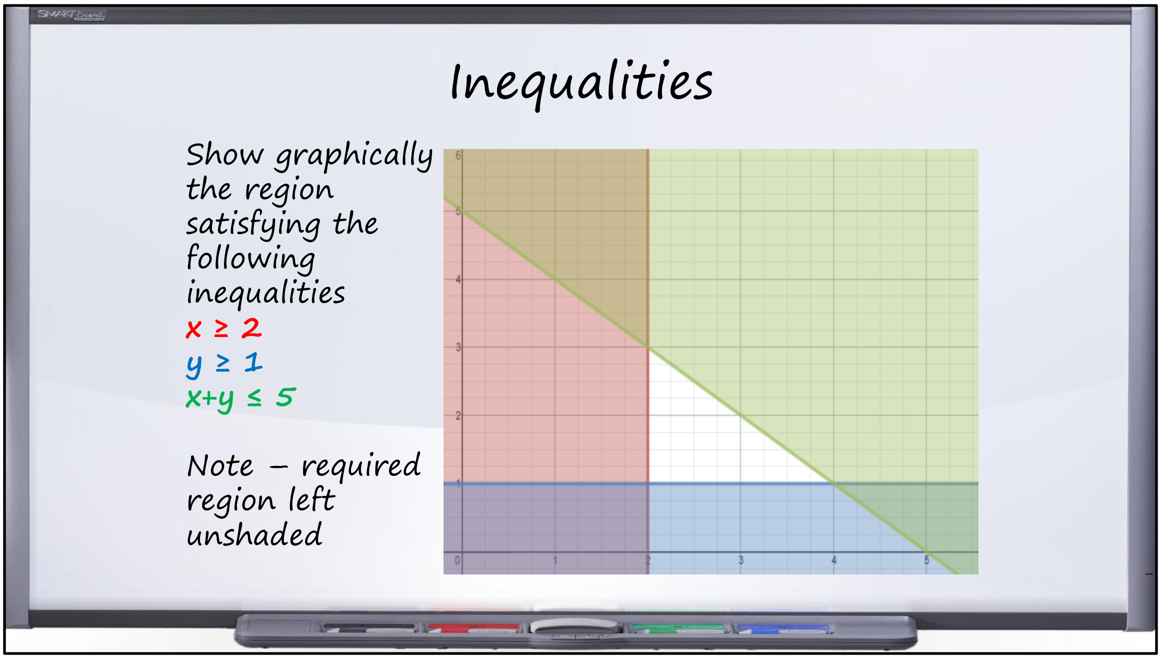
$f(g(x)) = f(x+3) = 2(x+3)^2$.

$2(x+3)^2 = 2(x^2+6x+9) = 2x^2 + 12x + 18$

$fg(x) = 2x^2 + 12x + 18$

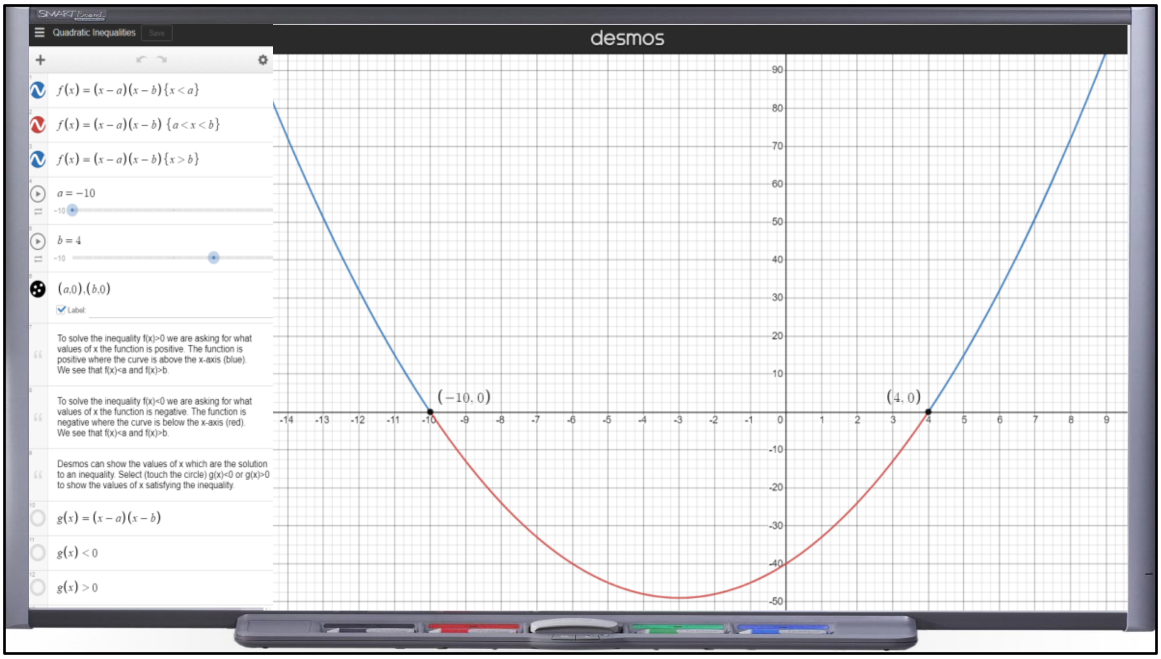
Question continued.

Just a simple use of colour but hopefully helpful.



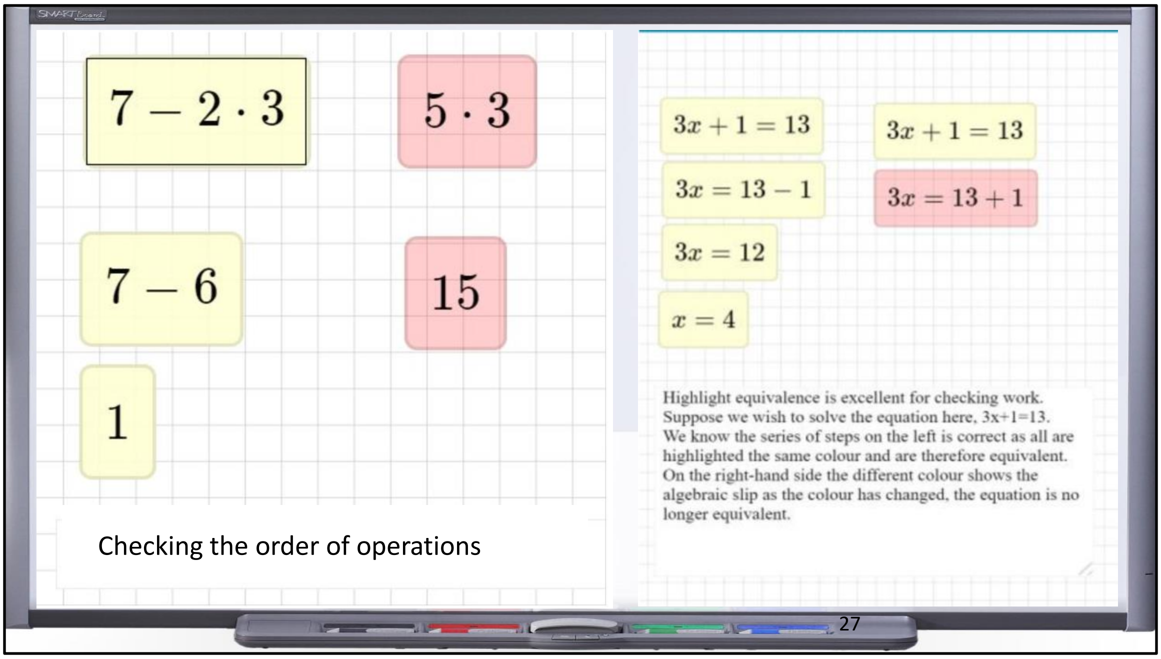
A typical GCSE question, associating colour of region with inequalities on Desmos.

You could use Autograph here which allows you to specify which region is to be shaded.



Staying with inequalities, I find it helpful to use colour as I have here to emphasise where the function has values above and below zero.

I find the ability to add notes in Desmos useful, I have sent many a page like this to my distance learners.



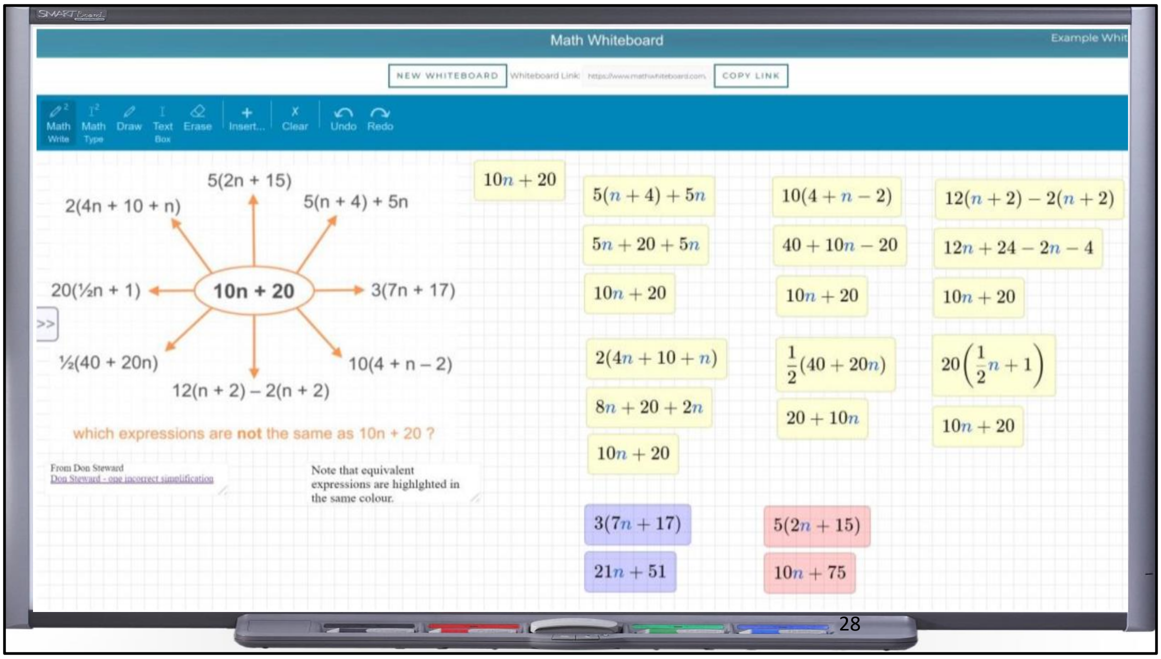
Now on Math Whiteboard (all free) (not to be confused with Matt Woodfine's brilliant Maths White Board).

Highlight equivalence feature is excellent for checking work.

Suppose we wish to solve the equation here, 3x+1=13.
We know the series of steps on the left is correct as all are highlighted the same colour and are therefore equivalent.

On the right-hand side, solving an equation, each time we enter an equivalent expression the colour remains unchanged.

The different colour, pink, shows the algebraic slip as the colour has changed, the equation is no longer equivalent.

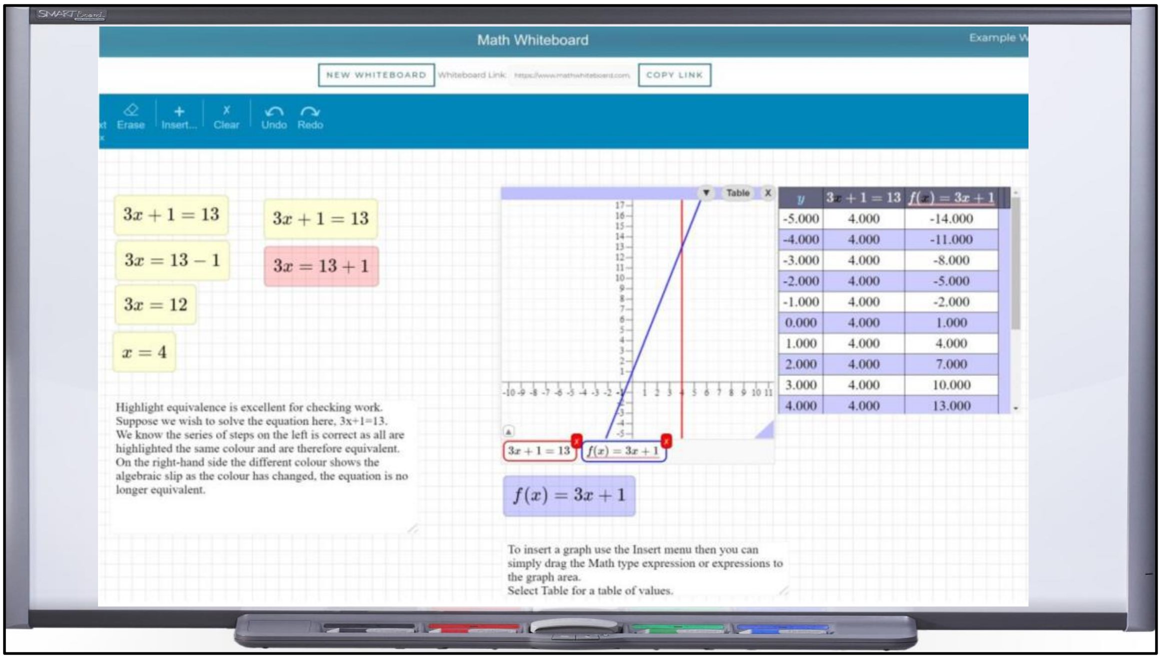


Possible to insert images into Math Whiteboard as I have here from Don Steward's Median.

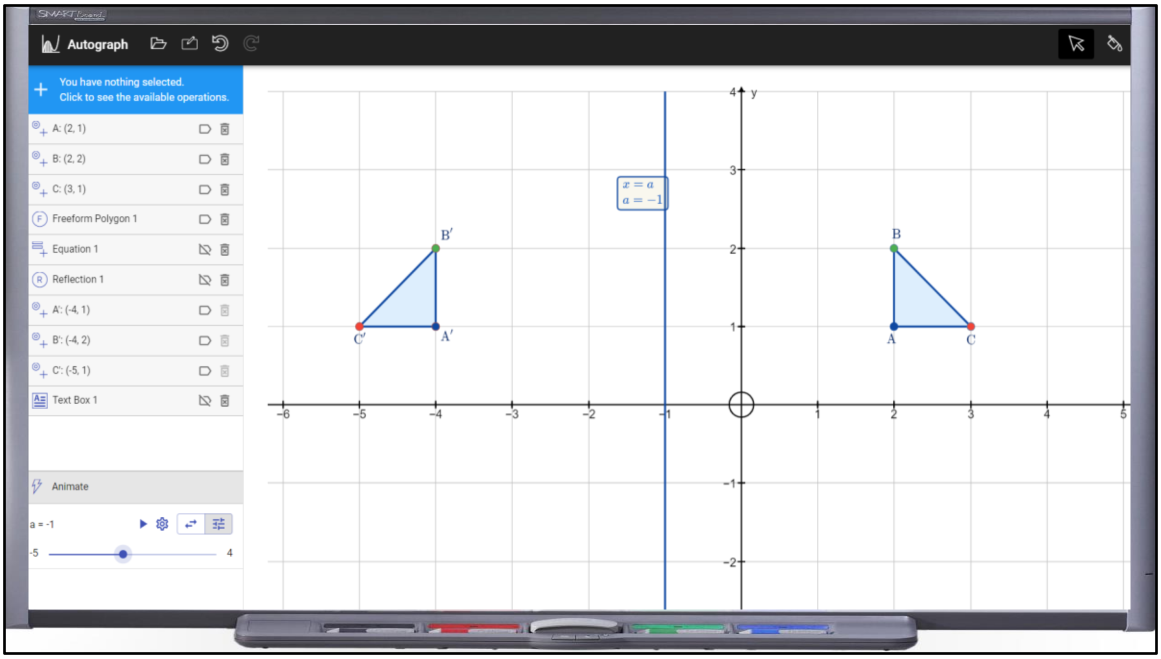
Task where students must identify which expressions are not the same as 10n+20

Highlight equivalence again ideal to check correct working at each step.

Equivalent expressions highlighted
(all variables blue)



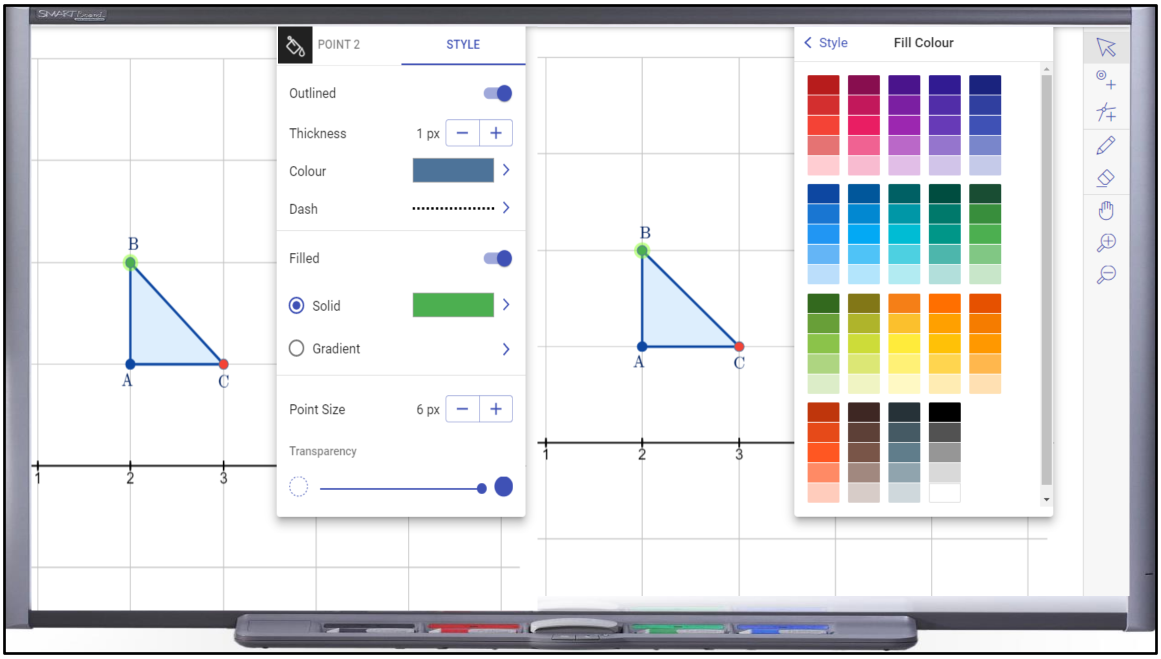
Also possible to easily insert a graphical representation.



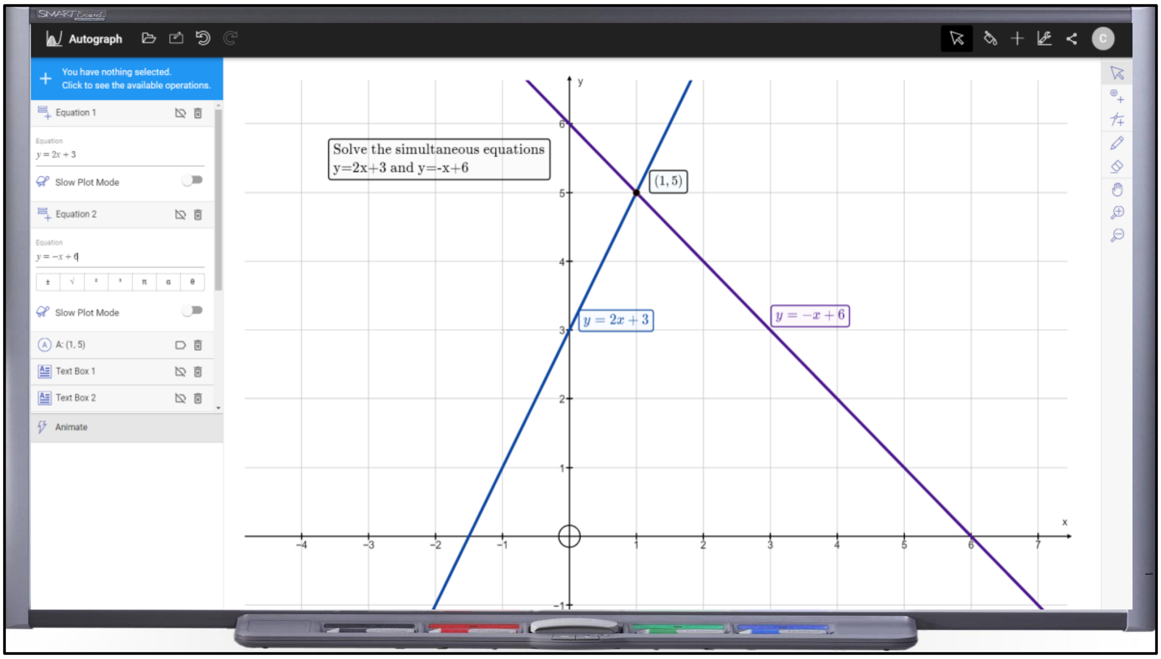
Autograph web, Saturday 10th July 2021 an important day – the launch of Autograph's web version at La Salle's Maths Conference #26.
So now it is possible to share a page with students which is excellent.

What I like about Autograph is the ability to style elements, it is simple to change colours of points, lines or areas for example.

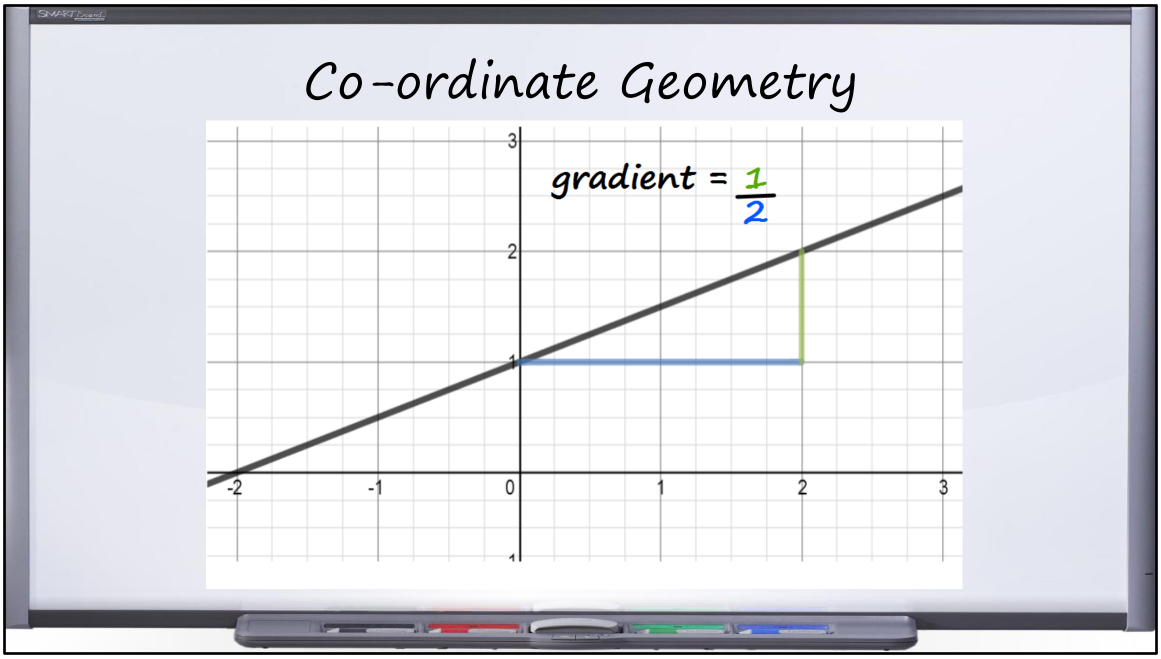
Note here for the reflections I have used the same colour for a point and its corresponding image.



Having selected a point, I can then choose to style it. Note the colour palette available, a great deal of choice.

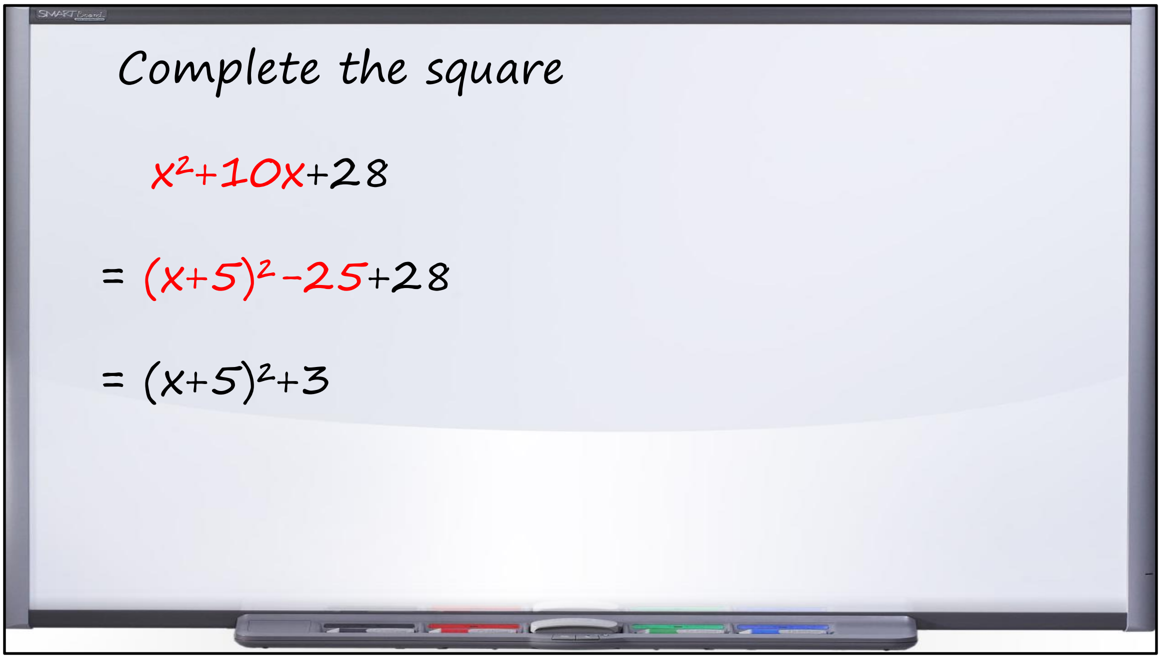


Here as you can see I have created labels to match each line. The ability to add text boxes and style them as you wish means we can present very useful pages in Autograph.



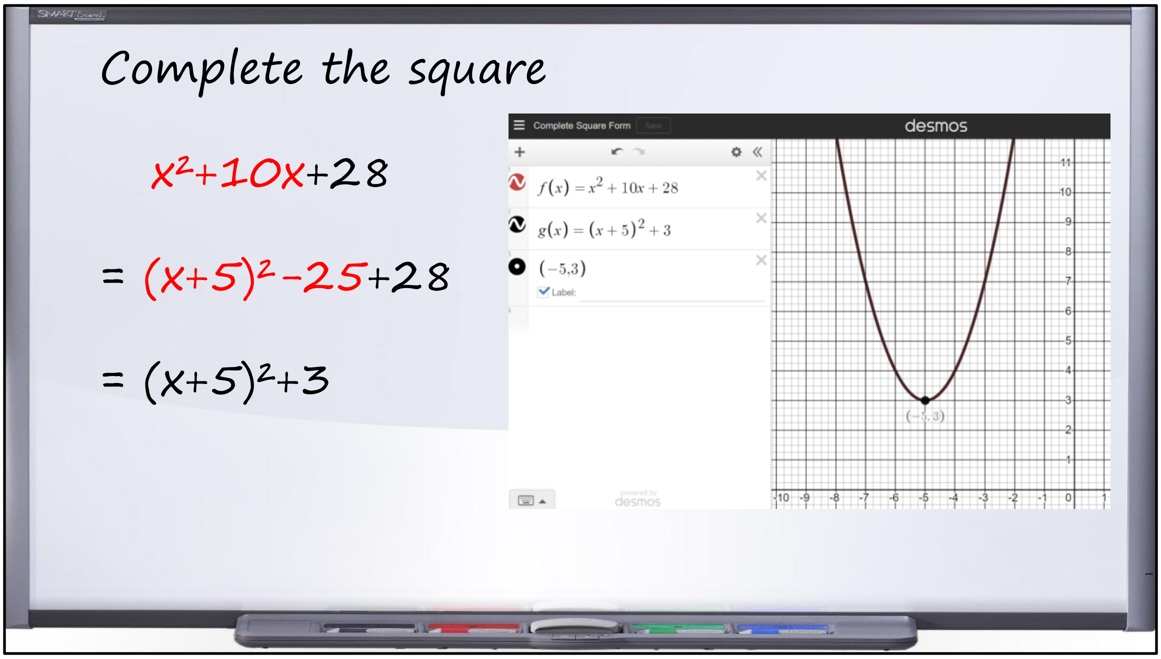
A few more illustrations ...

Very simply colouring a diagram to explain the numerical values of the gradient calculation.



A common theme with using colour for me is to show equivalence which you can see is what I am up to here.

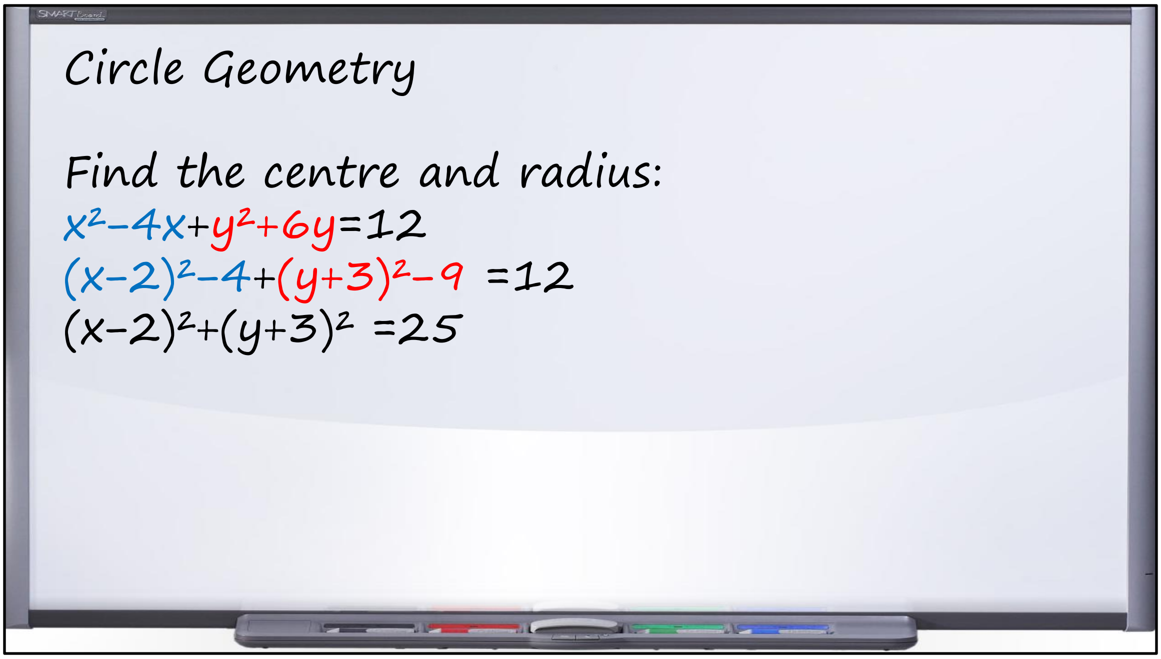
Let’s deal with that x squared +10x
x squared +10x is the same as….



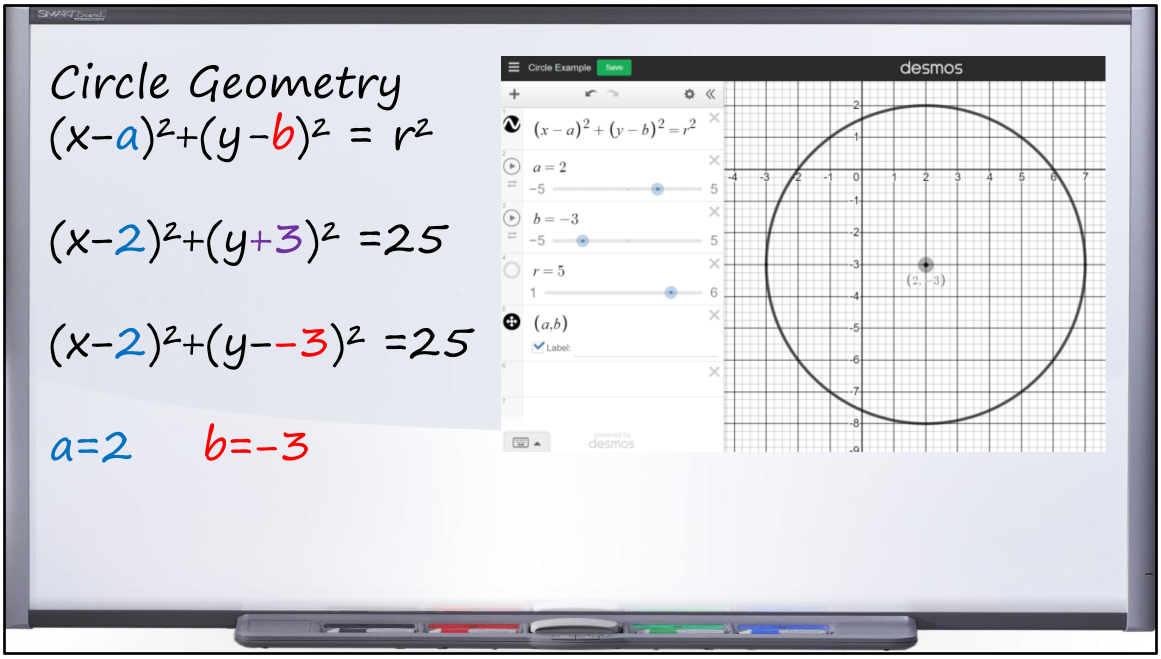
…and of course we check.

A nice way to check equivalence on Desmos – type in both and the second form sits on top of the original.

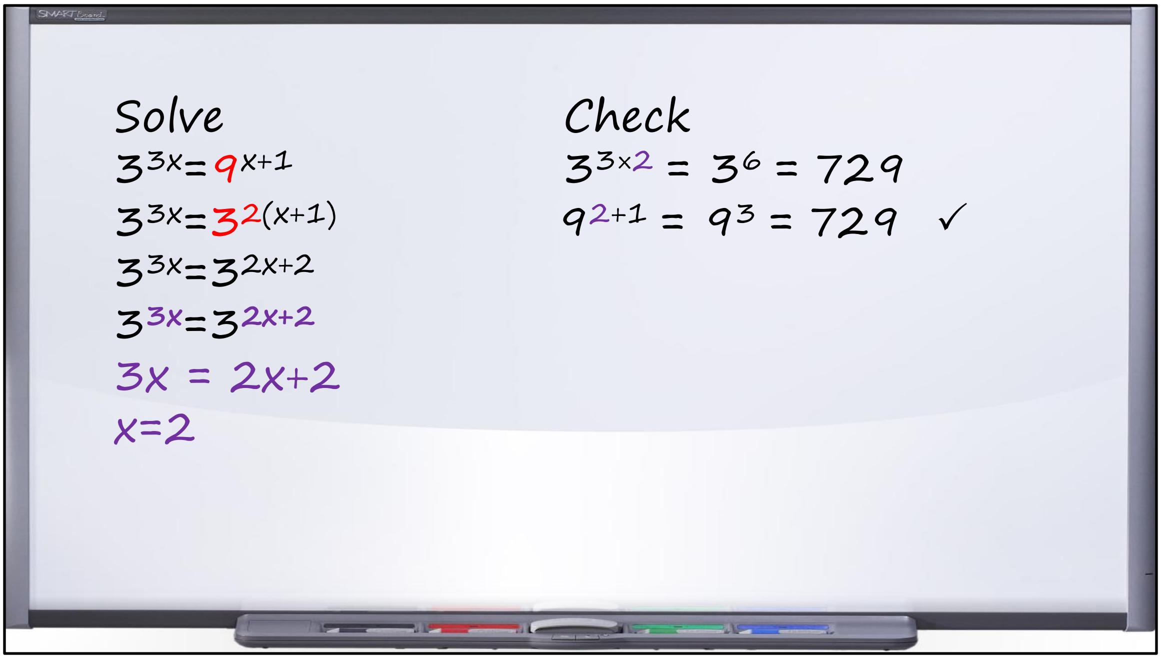
Link Algebra and graphical representation wherever possible.



Extend to circles…

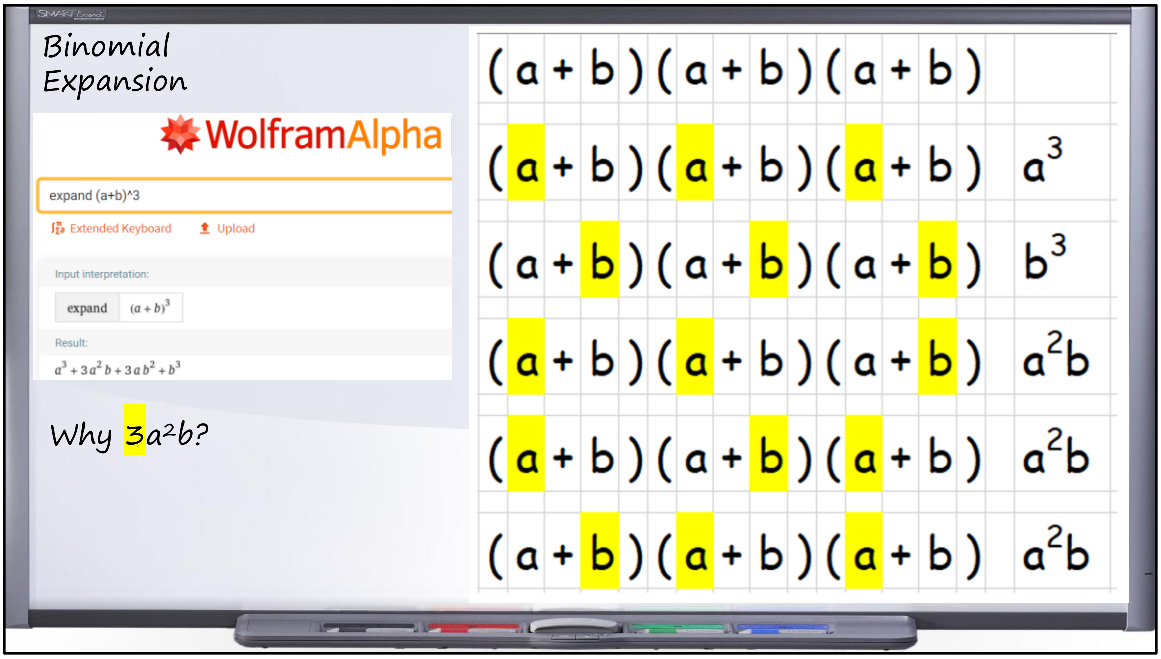


Extend to circles…
And generalise



Solving equations….showing equivalence again with the 9 and the 3 squared.

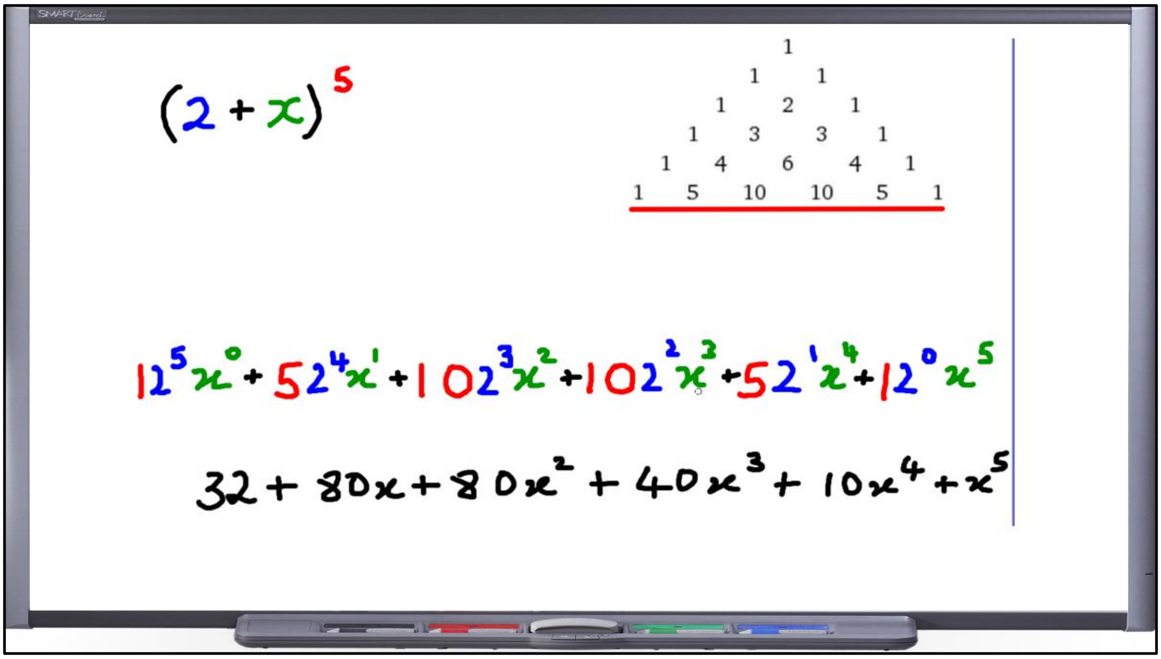
And then in the later step with the powers 3x and 2x+2



A level – Binomial expansion.
This was me explaining where the coefficient of 3 came from with a^2b.

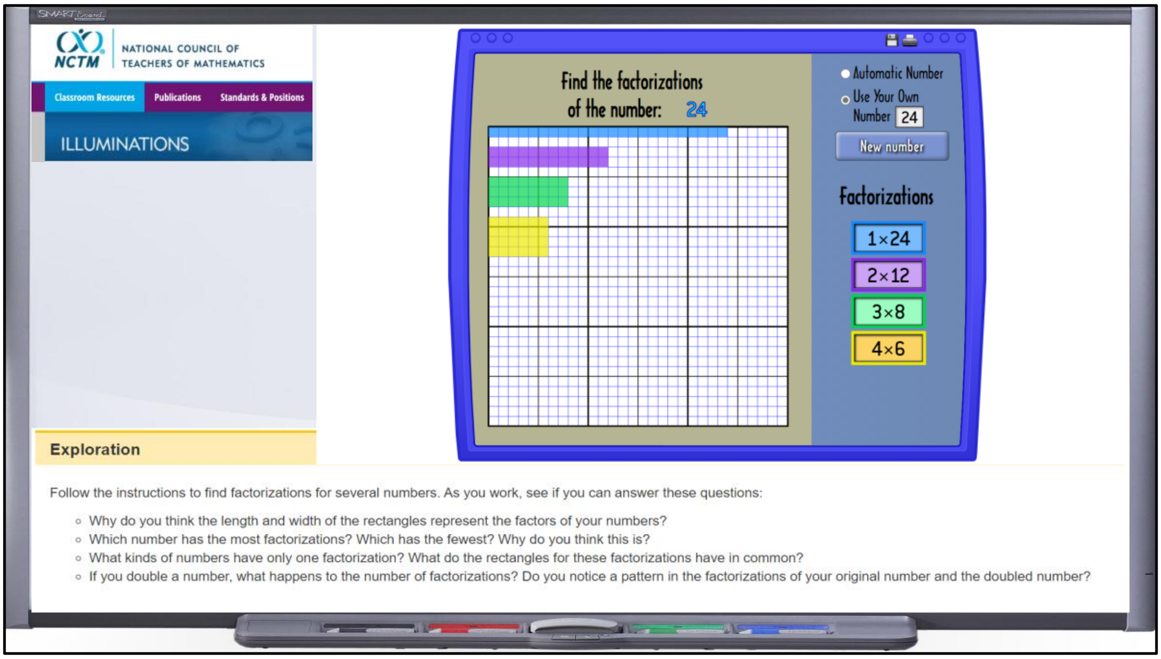
Simple highlighting to show there are three ways of generating a^2b

Excel used to line everything up.



Teaching the Binomial expansion would always see me at the board with 3 different colour pens.
Write up the coefficients then decreasing powers of 2 and finally increasing powers of x.

The technique is to space out the terms across the board so you don't run out of room!

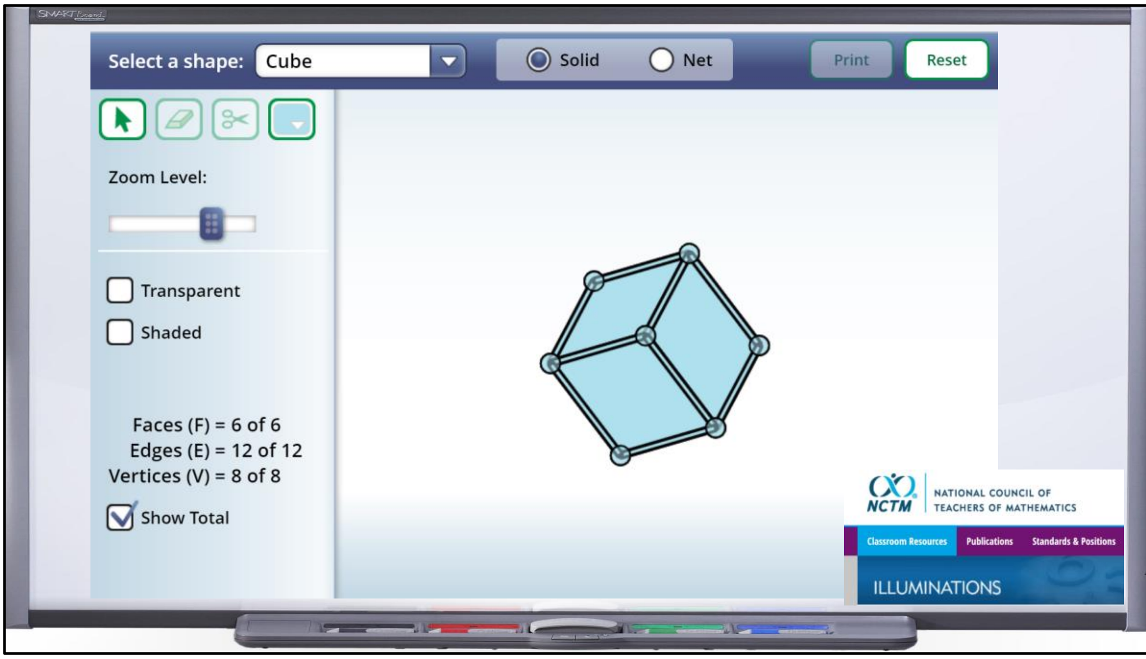


Numerous interactives from NCTM with many tasks for students.

Good use of colour with again the text colour coded as the diagram so we see for example in purple the 2x12 rectangle on the left and its matching factorisation on the right.

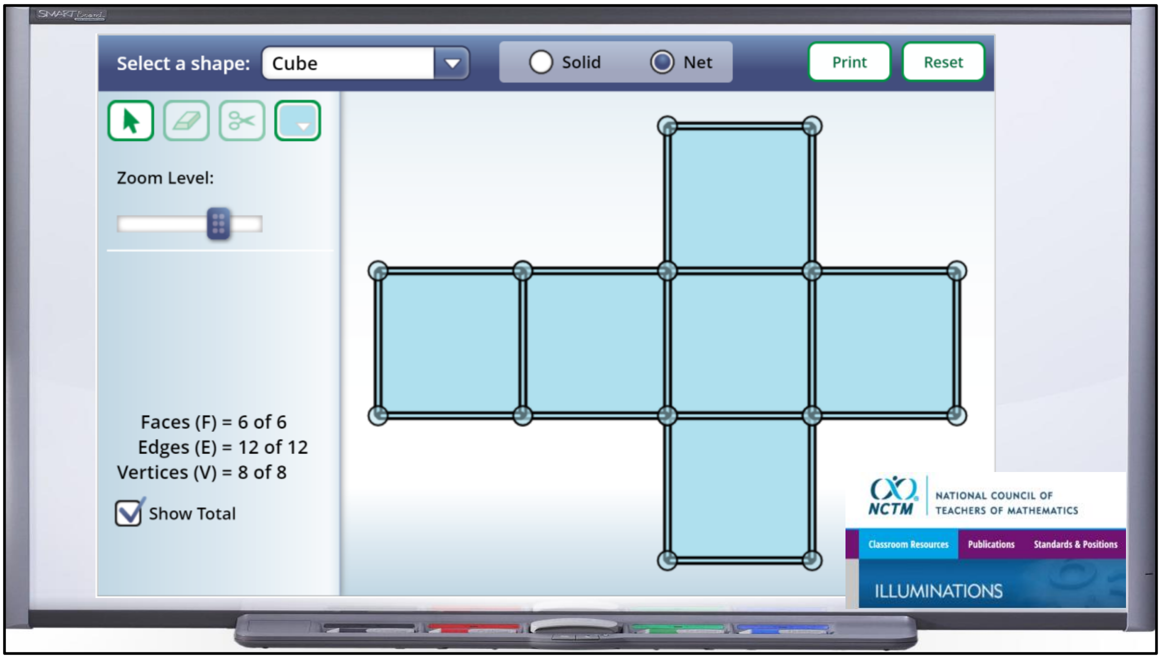
Obviously we cannot just let students loose with the technology, they need structure / guidance.

Suggestions are given and questions as you can see here.

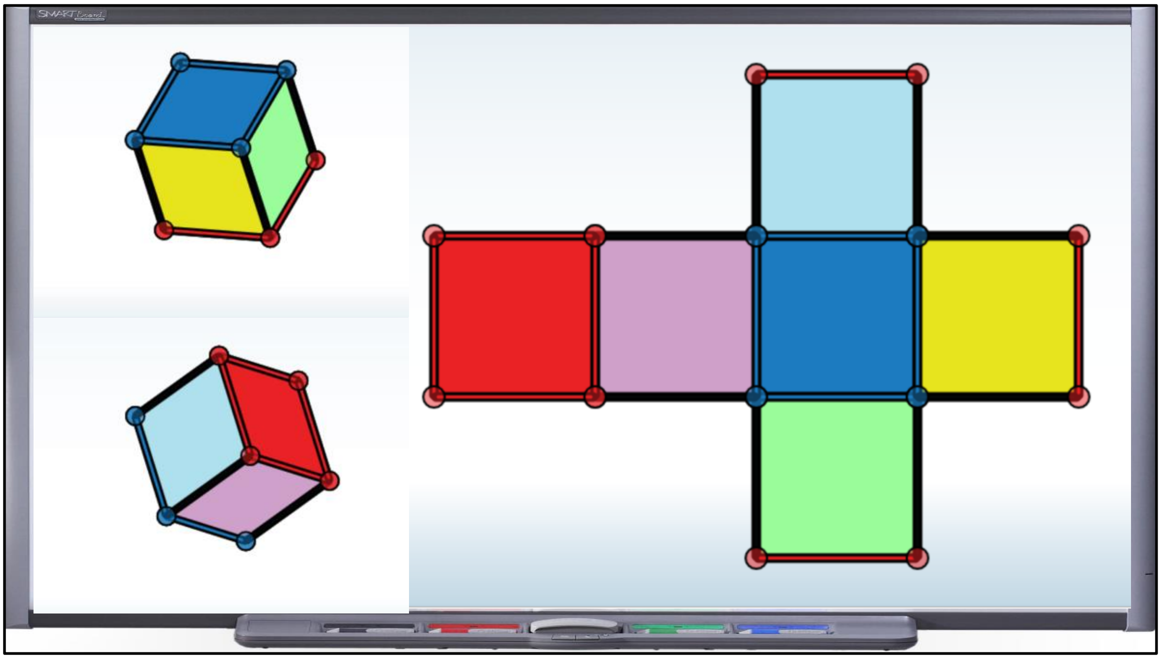


Geometry, NCTM Illuminations again.

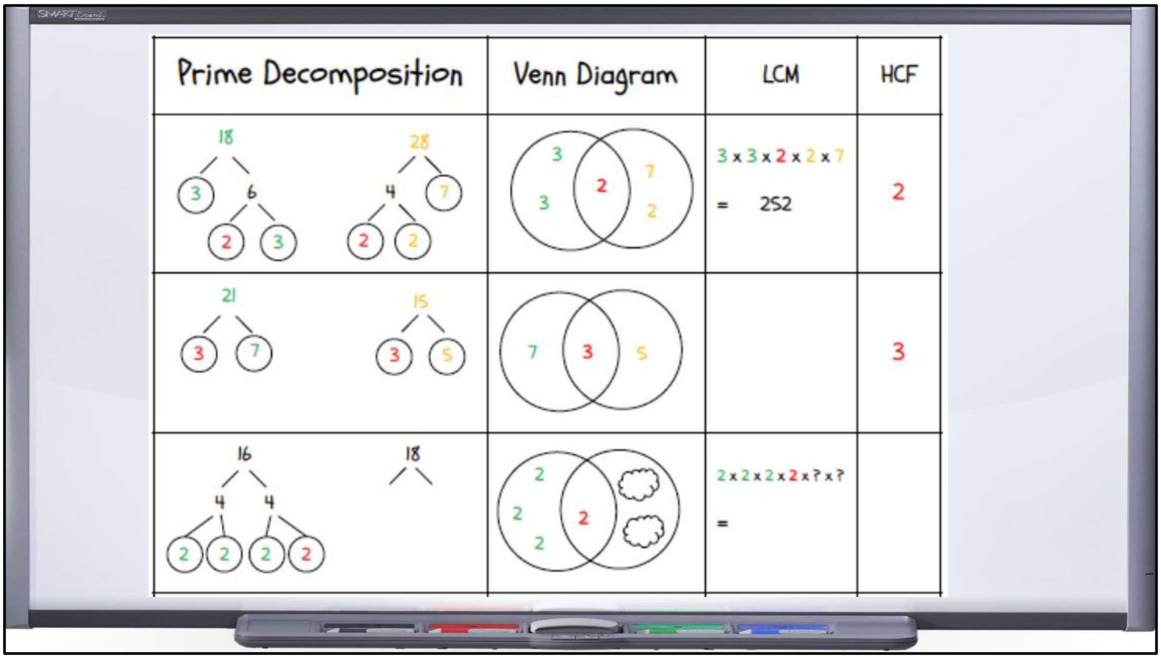
Here students can explore the relationship between faces edges and vertices, as well as see the net.



You can colour individual elements; faces, edges and vertices.

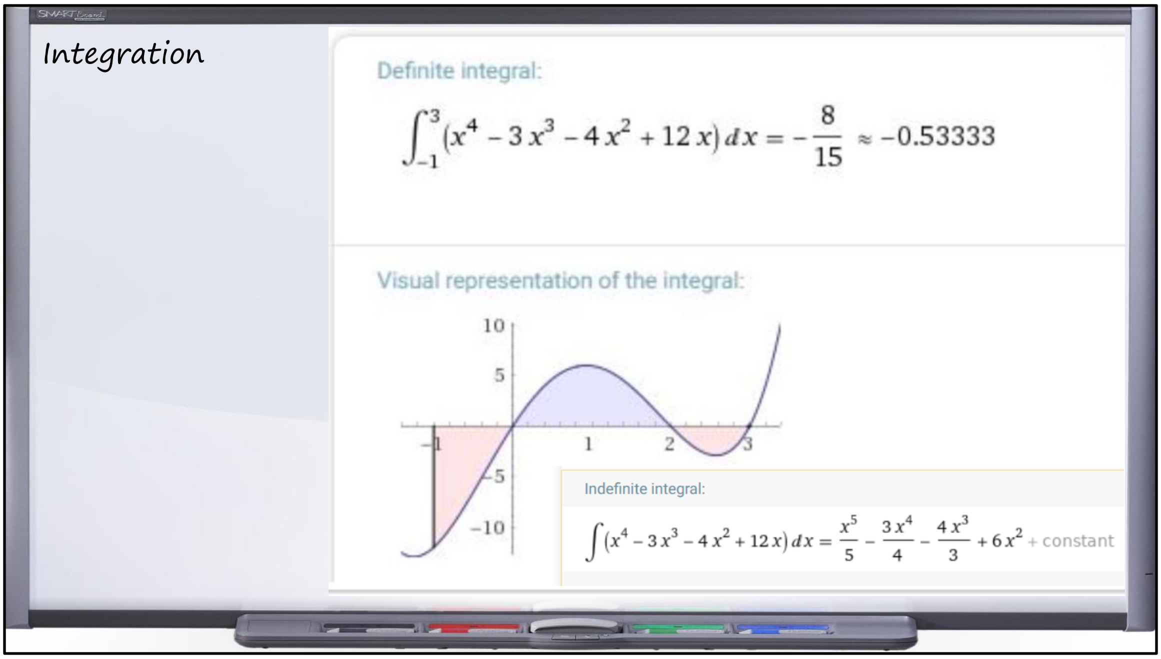


It is possible to rotate the 3D view of the model.



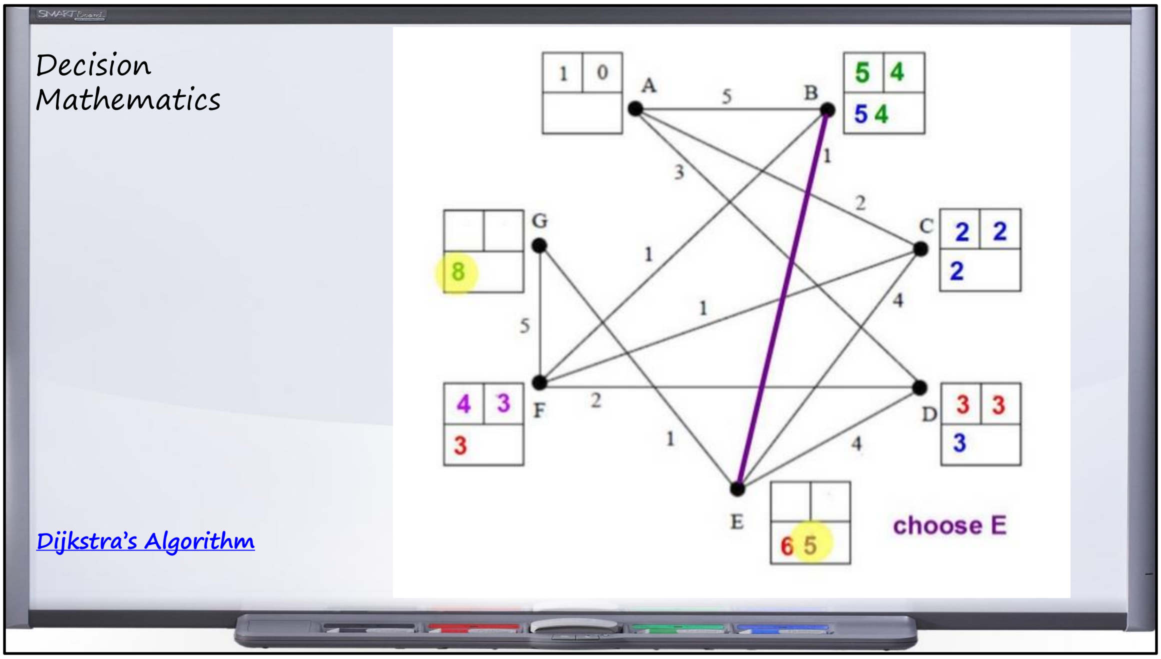
This task from Chris McGranes's Starting Points Maths Completion task…

A very nice use of colour here…
Note the consistency of colour between the different representations.

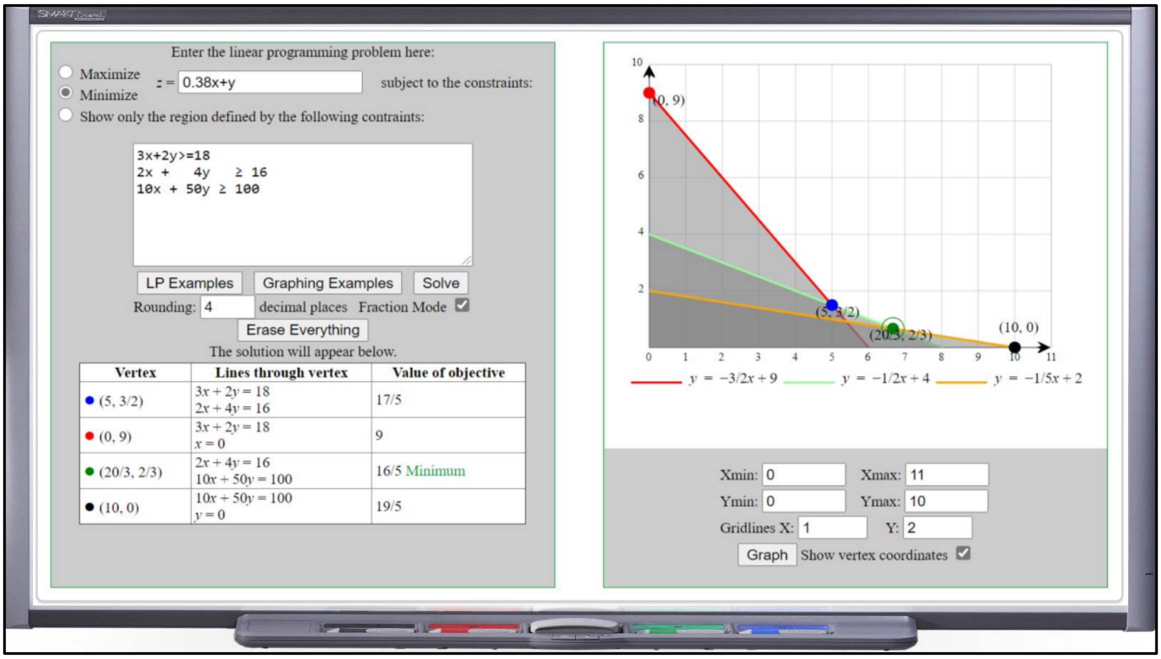


More A level
WolframAlpha for a lovely visual representation….

I frequently use graphing software, also WolframAlpha to illustrate the work we are doing.



Dijkstra's Algorithm.
My Year 12 students had just completed a Decision Mathematics mock examination. Preparing solutions to questions causing the most problems made me realise how often I use colour and highlighting in my explanations of exam questions where I think this helps clarity. This is from a question on Dijkstra's algorithm for finding shortest paths in network. I changed colour each time a new vertex was chosen.



This lovely linear programming solver has used a technique I have already mentioned – colouring parts of a diagram and then using the same colours in the explanation.
Here we can see a vertex tour.

Helps interpret what you are seeing more easily.
Draws your attention.

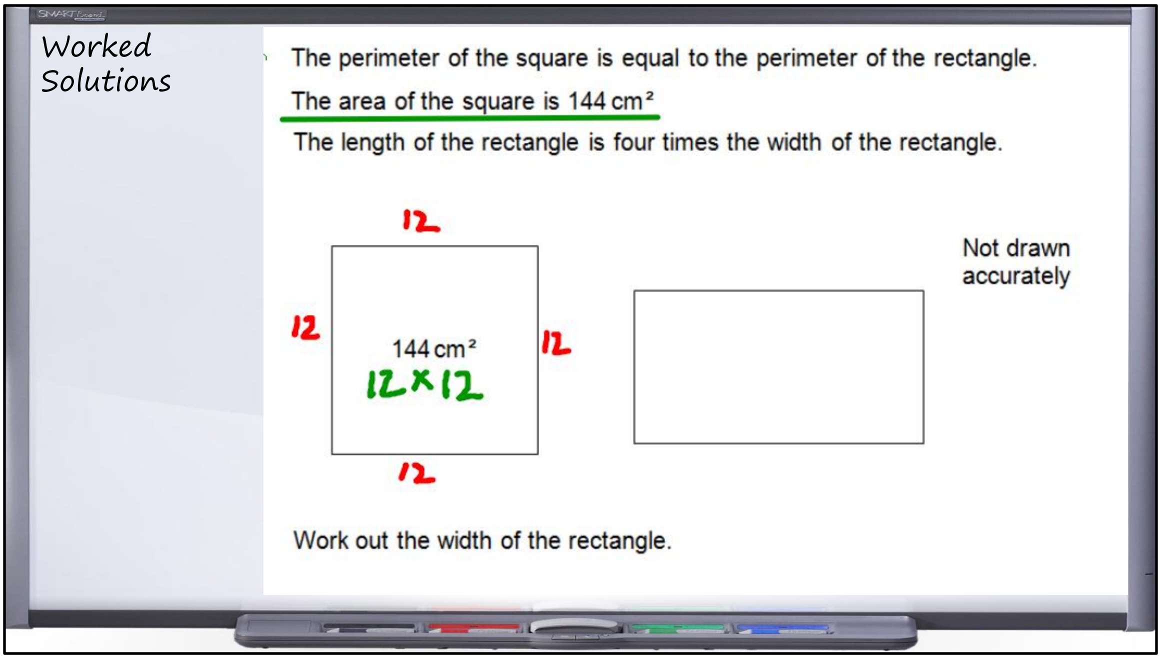
Worked Solutions

The use of colour can help clarity in online worked solutions for students.

Interestingly, some students prefer a series of still images with no sound to videos as they can really dictate the pace themselves.

Some examples follow…

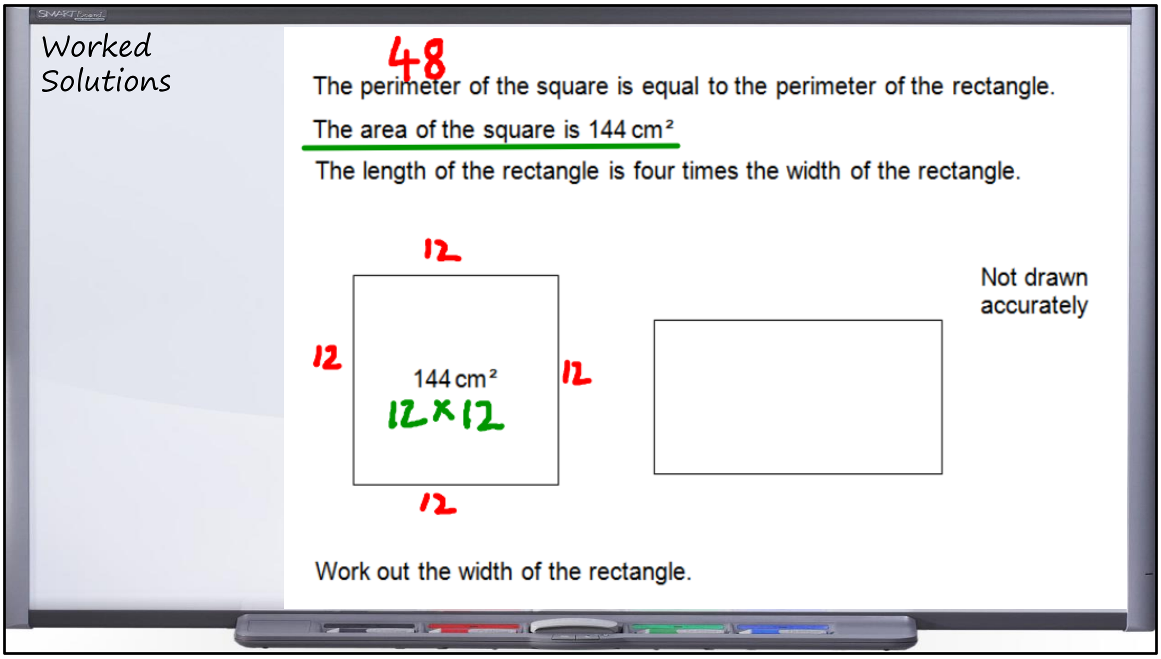
More worked solutions….



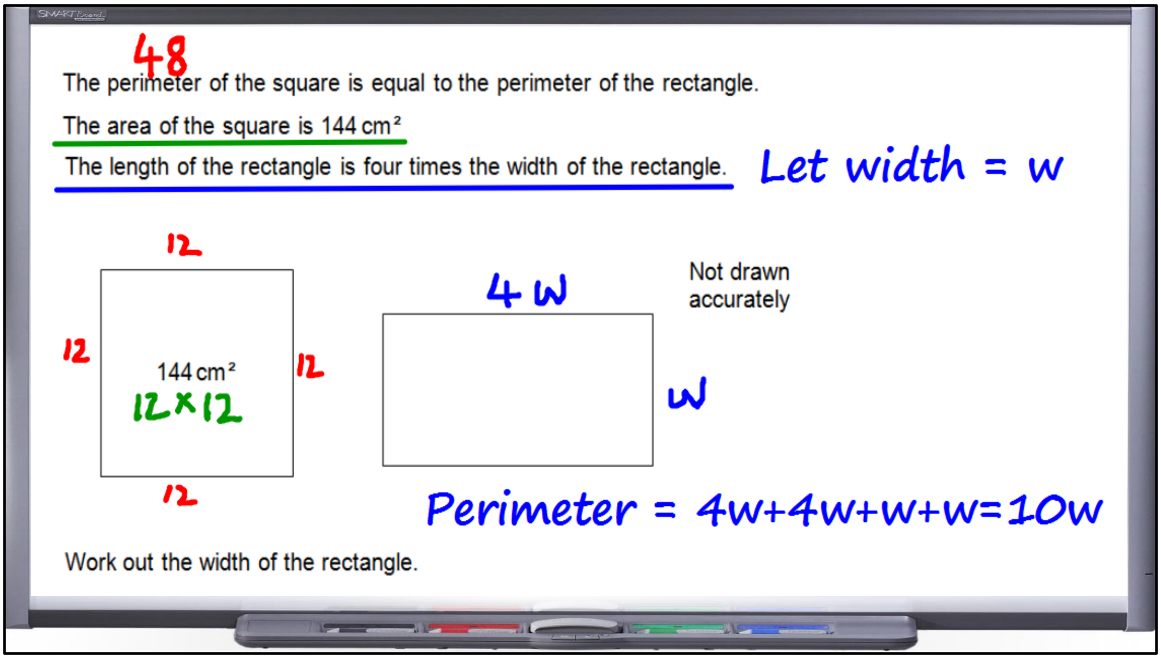
Typical of something I do a lot.
Underline parts of a question in colour and then translate that statement into symbols elsewhere in the explanation.

Typical GCSE question linking Geometry and Algebra.

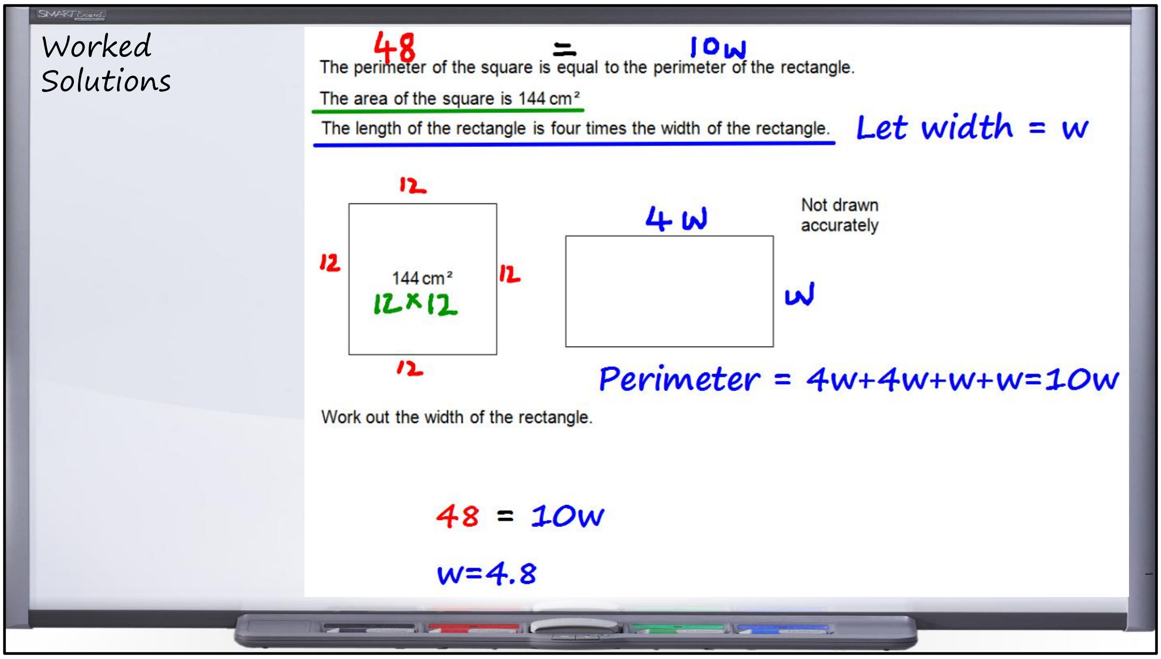
(Change that red/green!)



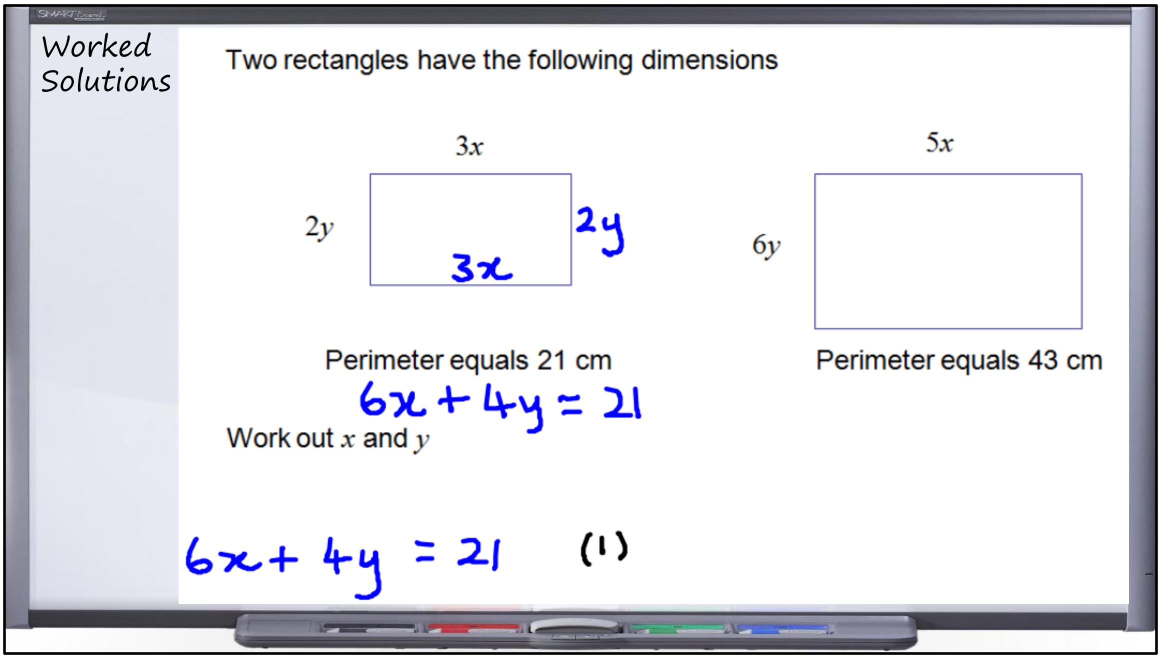
Two colours showing the area and perimeter



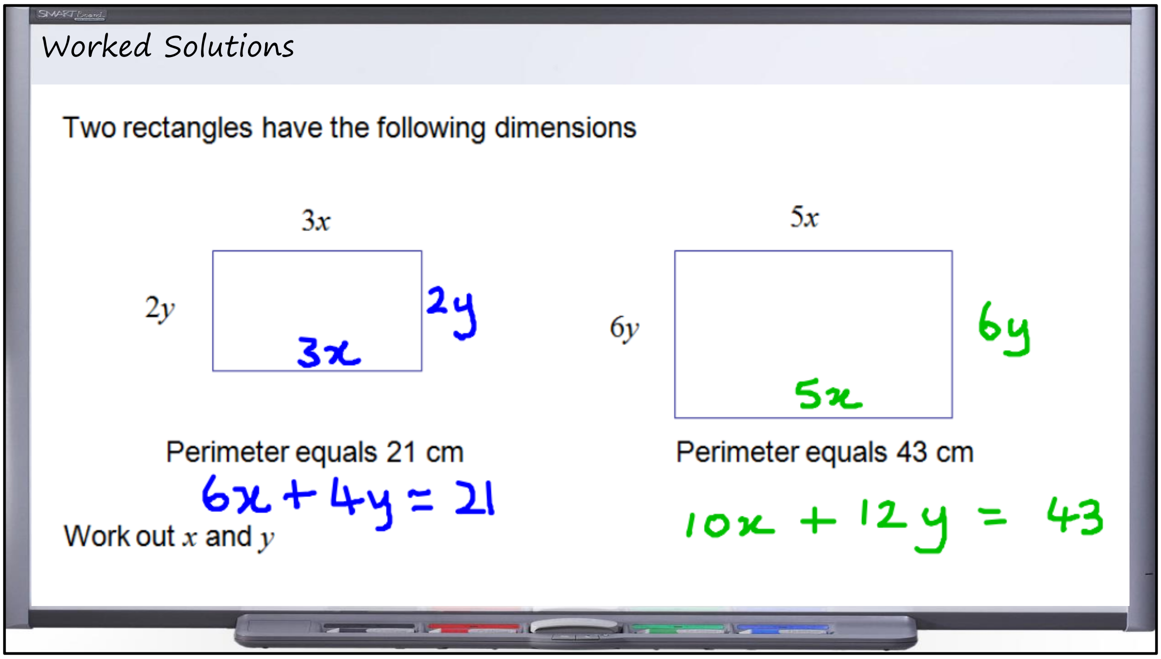
Third colour now introduced for the length of the rectangle.



Annotating the question using colour and modelling for students.



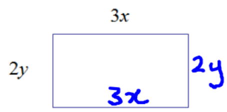
Further examples…



A simple use of colour to illustrate where each equation comes from.

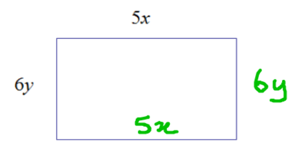
Worked Solutions

Two rectangles have the following dimensions



Perimeter equals 21 cm

$6x + 4y = 21$



Perimeter equals 43 cm

$10x + 12y = 43$

Work out x and y

$6x + 4y = 21 \quad (1)$

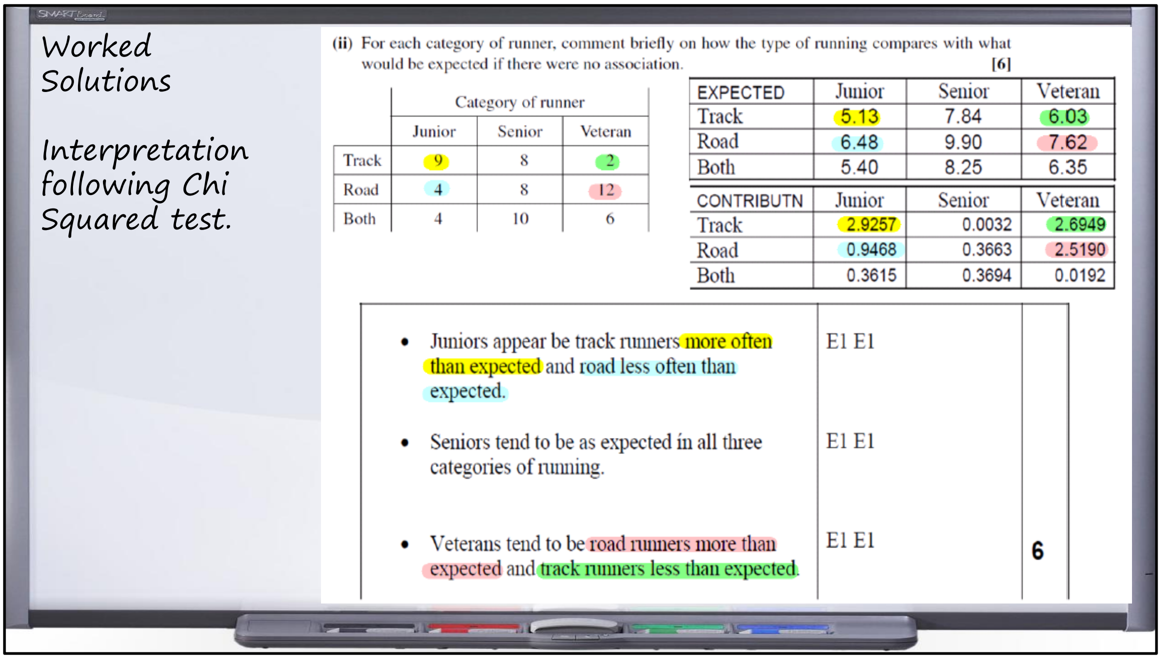
$10x + 12y = 43 \quad (2)$

$18x + 12y = 63 \quad (1) \times 3$

$10x + 12y = 43 \quad (2)$

$8x = 20 \qquad x = 2.5$

13	63-18x = 43 –10x	M1
	4y =21-15	M1
	x = 2.5 y = 1.5	A1



Worked Solutions

Interpretation following Chi Squared test.

(ii) For each category of runner, comment briefly on how the type of running compares with what would be expected if there were no association. **[6]**

	Category of runner		
	Junior	Senior	Veteran
Track	9	8	2
Road	4	8	12
Both	4	10	6

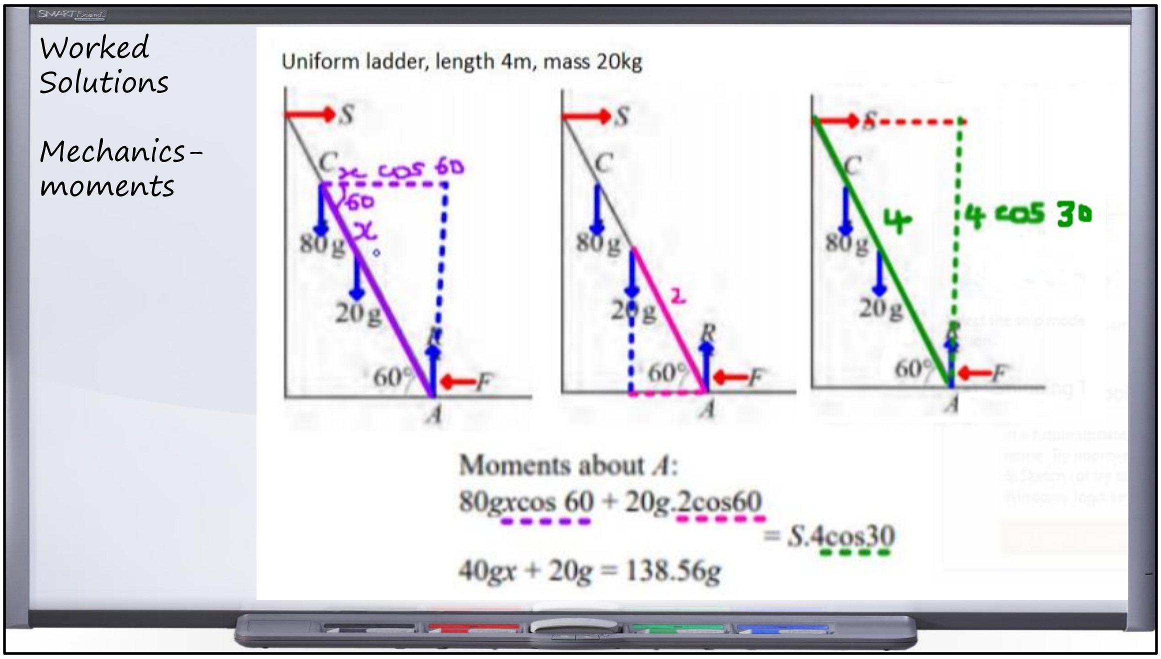
EXPECTED	Junior	Senior	Veteran
Track	5.13	7.84	6.03
Road	6.48	9.90	7.62
Both	5.40	8.25	6.35

CONTRIBUTN	Junior	Senior	Veteran
Track	2.9257	0.0032	2.6949
Road	0.9468	0.3663	2.5190
Both	0.3615	0.3694	0.0192

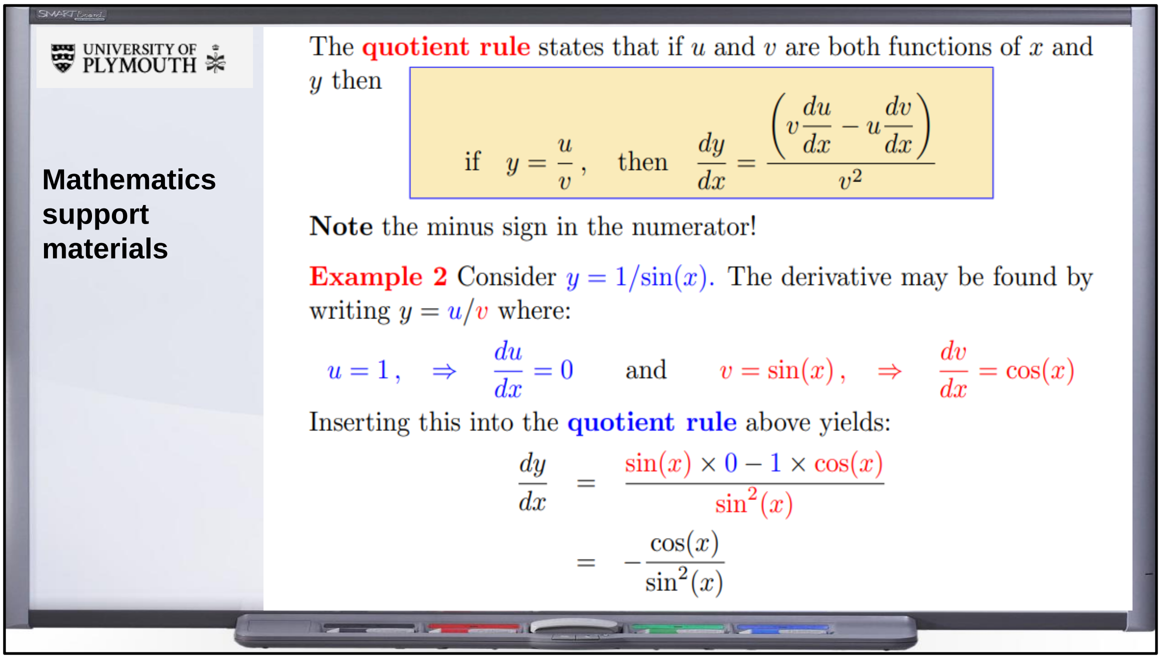
• Juniors appear be track runners more often than expected and road less often than expected.	E1 E1	
• Seniors tend to be as expected in all three categories of running.	E1 E1	
• Veterans tend to be road runners more than expected and track runners less than expected.	E1 E1	6

At A level I have so often found that when students perform a chi squared test, doing the test is not a problem, but they were losing marks on interpretation.

I have always reminded them to look at all the above, the actual results, the expected results and the contributions to the text statistic. Highlighting with colour makes it much easier to compare the figures.

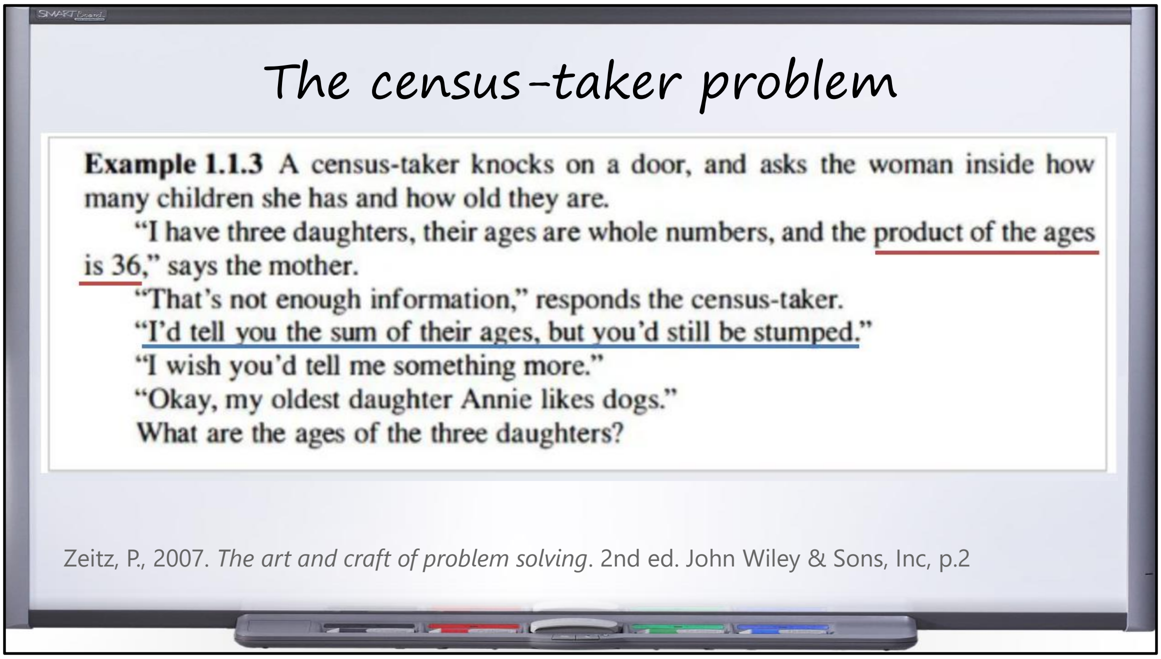


Moments in Mechanics….



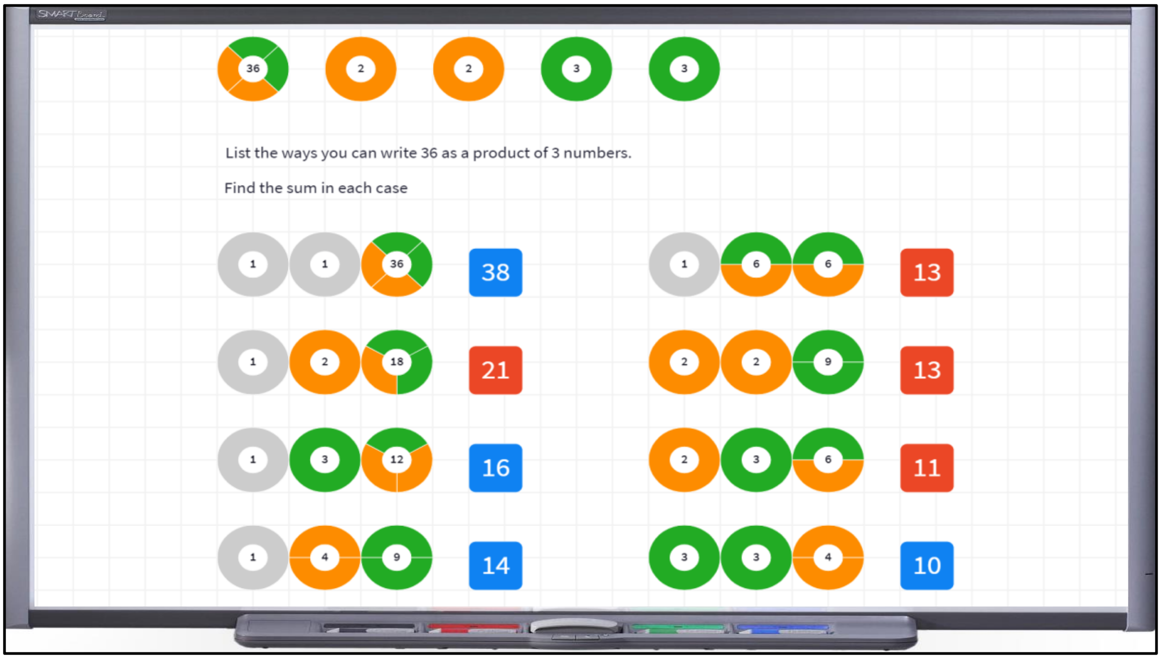
University of Plymouth Mathematics support materials.

Notice the use of colour to emphasise the numerator and denominator.



A final problem…

Pause to read.

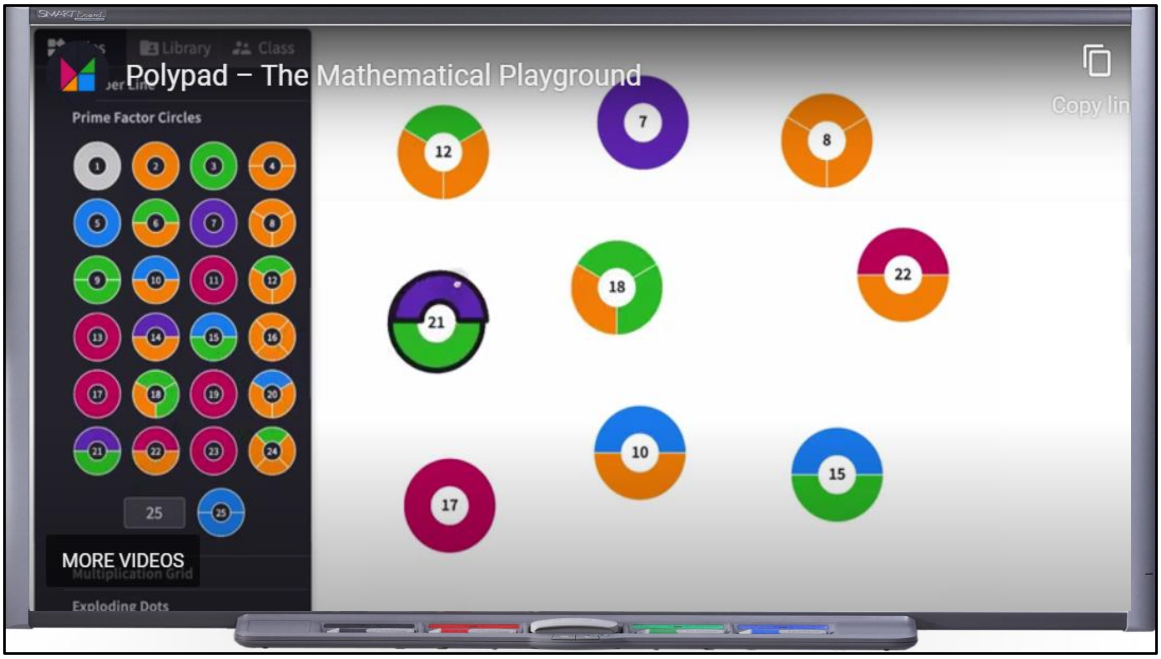


The **product of the ages is 36**, so only a few possible triples of ages.
Aha! Now we see what is going on.
The mother's second statement ("**I'd tell you the sum of their ages, but you 'd still be stumped**") gives us valuable information. It tells us that the ages are either (1 ,6,6) or (2,2, 9), for in all other cases, knowledge of the sum would tell us unambiguously what the ages are !

The final clue now makes sense; it tells us that there is an oldest daughter, eliminating the triple (1 ,6,6) . The daughters are thus 2, 2 and 9 years old.

Using Mathigon to illustrate - Using Mathigon – Polypad Prime

Factor Circles.

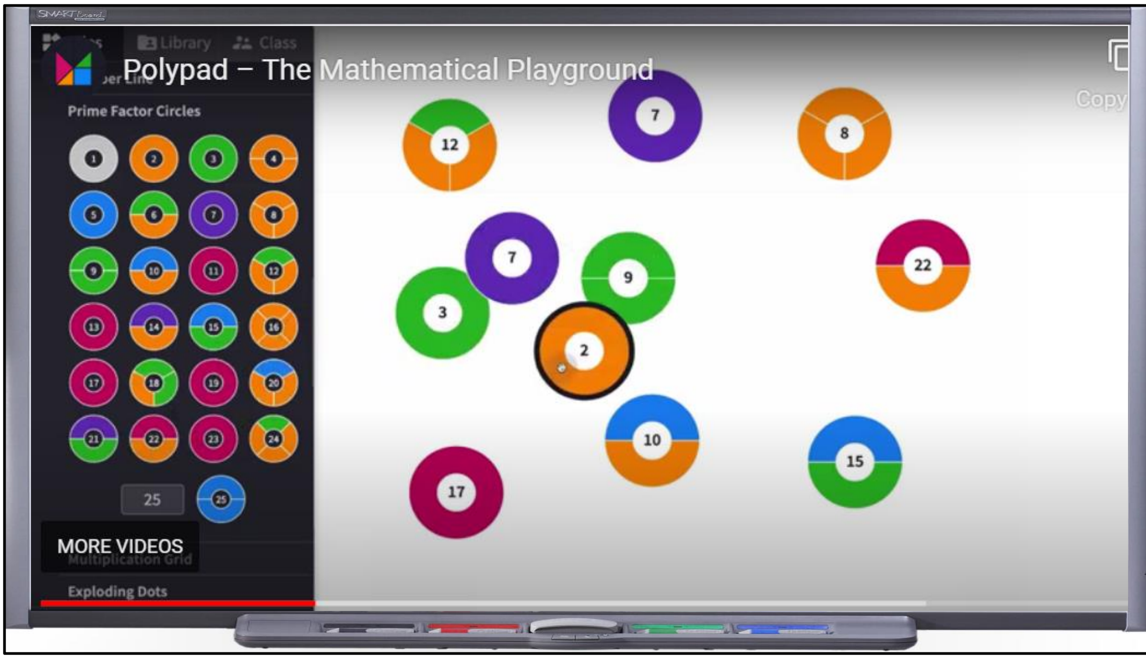


Using Mathigon – Polypad Prime Factor Circles.

Look at the use of colour, look at the left hand side, we have a green 3 and a purple 7.

Note 21 = 3x7
And 18 = 3x3x2

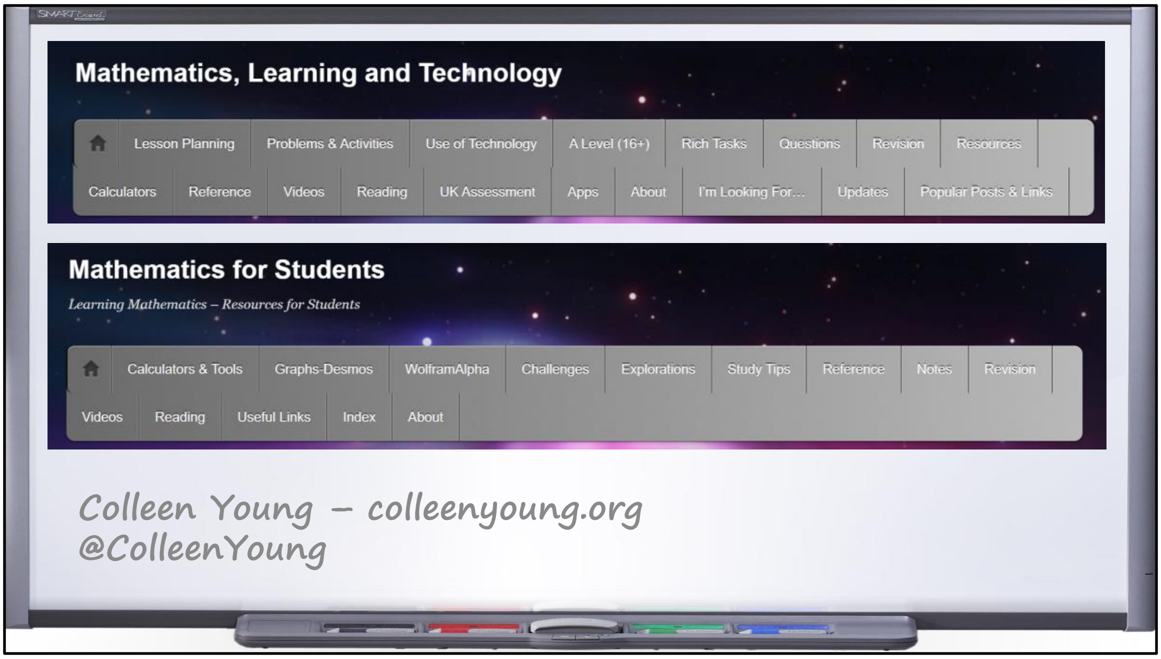
Drag the purple from 21…



Select and drag the purple part of 21 and we have 3x7.

Similarly 18 is 9x2 – we could further separate the 9 into 3x3.

Note you can make your own number circles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