Department for Education

Mathematics

AS and A level content

April 2016

Content for mathematics AS and A level for teaching from 2017

Introduction

1. AS and A level subject content sets out the knowledge, understanding and skills common to all AS and A level specifications in mathematics.

Purpose

2. A level mathematics provides a framework within which a large number of young people continue the subject beyond GCSE level. It supports their mathematical needs across a broad range of other subjects at this level and provides a basis for subsequent quantitative work in a very wide range of higher education courses and in employment. It also supports the study of AS and A level further mathematics.

3. A level mathematics builds from GCSE level mathematics and introduces calculus and its applications. It emphasises how mathematical ideas are interconnected and how mathematics can be applied to model situations mathematically using algebra and other representations, to help make sense of data, to understand the physical world and to solve problems in a variety of contexts, including social sciences and business. It prepares students for further study and employment in a wide range of disciplines involving the use of mathematics.

4. AS mathematics, which can be co-taught with the A level as a separate qualification, is a very useful qualification in its own right. It consolidates and develops GCSE level mathematics and supports transition to higher education or employment in any of the many disciplines that make use of quantitative analysis, including those involving calculus.

Aims and objectives

5. AS and A level specifications in mathematics must encourage students to:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general

- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development

Subject content

Background knowledge

6. AS and A level mathematics specifications must build on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics for first teaching from 2015. The knowledge and skills required for AS mathematics are shown in the following tables in bold text within square brackets.

Overarching themes

7. A level specifications in mathematics must require students to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below.

OT1 Mathematical argument, language and proof

AS and A level mathematics specifications must use the mathematical notation set out in appendix A and must require students to recall the mathematical formulae and identities set out in appendix B.

	Knowledge/Skill
OT1.1	**[Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable]**
OT1.2	**[Understand and use mathematical language and syntax as set out in the content]**
OT1.3	**[Understand and use language and symbols associated with set theory, as set out in the content]** **[Apply to solutions of inequalities]** and probability
OT1.4	Understand and use the definition of a function; domain and range of functions
OT1.5	**[Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics]**

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	**[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]**
OT2.2	**[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]**
OT2.3	**[Interpret and communicate solutions in the context of the original problem]**
OT2.4	Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
OT2.5	**[Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions]**, including those obtained using numerical methods
OT2.6	**[Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle]**
OT2.7	**[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics]**

OT3 Mathematical modelling

	Knowledge/Skill
OT3.1	**[Translate a situation in context into a mathematical model, making simplifying assumptions]**
OT3.2	**[Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)]**
OT3.3	**[Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)]**
OT3.4	**[Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate]**
OT3.5	**[Understand and use modelling assumptions]**

Use of technology

8. The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics. Calculators used must include the following features:

- an iterative function
- the ability to compute summary statistics and access probabilities from standard statistical distributions

Use of data in statistics

9. AS and A level mathematics specifications must require students to:

- become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored)
- use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
- interpret real data presented in summary or graphical form
- use data to investigate questions arising in real contexts

10. Specifications should require students to explore the data set(s), and associated contexts, during their course of study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data. Specifications should also require students to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions, as detailed in paragraph 8.

Detailed content statements

11. A level specifications in mathematics must include the following content. This, assessed in the context of the overarching themes, represents 100% of the content.

12. Content required for AS mathematics is shown in bold text within square brackets. This, assessed in the context of the AS overarching themes, represents 100% of the AS content.

A Proof

	Content
A1	**[Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion]** **[Disproof by counter example]** Proof by contradiction (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar proofs)

B Algebra and functions

	Content
B1	**[Understand and use the laws of indices for all rational exponents]**
B2	**[Use and manipulate surds, including rationalising the denominator]**
B3	**[Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown]**
B4	**[Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation]**
B5	**[Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions]** **[Express solutions through correct use of 'and' and 'or', or through set notation]** **[Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically]**
B6	**[Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem]**

	Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only)
B7	**[Understand and use graphs of functions; sketch curves defined by simple equations including polynomials]**, the modulus of a linear function, **[** $y=\frac{a}{x}$ **and** $y=\frac{a}{x^2}$ **(including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations]** **[Understand and use proportional relationships and their graphs]**
B8	Understand and use composite functions; inverse functions and their graphs
B9	**[Understand the effect of simple transformations on the graph of** $y=\mathrm{f}(x)$ **including sketching associated graphs:** $y=a\mathrm{f}(x)$, $y=\mathrm{f}(x)+a$, $y=\mathrm{f}(x+a)$, $y=\mathrm{f}(ax)$**]**, and combinations of these transformations
B10	Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear)
B11	Use of functions in modelling, including consideration of limitations and refinements of the models

C Coordinate geometry in the (x,y) plane

	Content
C1	**[Understand and use the equation of a straight line, including the forms** $y-y_1=m(x-x_1)$ **and** $ax+by+c=0$**; gradient conditions for two straight lines to be parallel or perpendicular]** **[Be able to use straight line models in a variety of contexts]**
C2	**[Understand and use the coordinate geometry of the circle including using the equation of a circle in the form** $(x-a)^2+(y-b)^2=r^2$**; completing the square to find the centre and radius of a circle; use of the following properties:** • **the angle in a semicircle is a right angle** • **the perpendicular from the centre to a chord bisects the chord** • **the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point**
C3	Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms
C4	Use parametric equations in modelling in a variety of contexts

D Sequences and series

	Content
D1	**[Understand and use the binomial expansion of $(a+bx)^n$ for positive integer n; the notations n! and nCr; link to binomial probabilities]** Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\| < 1$. (proof not required)
D2	Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$; increasing sequences; decreasing sequences; periodic sequences
D3	Understand and use sigma notation for sums of series
D4	Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms
D5	Understand and work with geometric sequences and series including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation
D6	Use sequences and series in modelling

E Trigonometry

	Content
E1	**[Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$]** Work with radian measure, including use for arc length and area of sector
E2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta,\ \cos\theta \approx 1-\frac{\theta^2}{2},\ \tan\theta \approx \theta$ where θ is in radians
E3	**[Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity]** Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof
E4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent;

	understanding of their graphs; their ranges and domains
E5	**[Understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$]** **[Understand and use $\sin^2\theta + \cos^2\theta = 1$]**; $\sec^2\theta = 1 + \tan^2\theta$ and $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$
E6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$; understand geometrical proofs of these formulae Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
E7	**[Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle]**
E8	Construct proofs involving trigonometric functions and identities
E9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces

F Exponentials and logarithms

	Content
F1	**[Know and use the function a^x and its graph, where a is positive]** **[Know and use the function e^x and its graph]**
F2	**[Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications]**
F3	**[Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$]** **[Know and use the function $\ln x$ and its graph]** **[Know and use $\ln x$ as the inverse function of e^x]**
F4	**[Understand and use the laws of logarithms:** $\log_a x + \log_a y = \log_a(xy)$; $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$; $k\log_a x = \log_a x^k$ **(including, for example, $k = -1$ and $k = -\frac{1}{2}$)]**
F5	**[Solve equations of the form $a^x = b$]**
F6	**[Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y]**
F7	**[Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models]**

G Differentiation

	Content
G1	**[Understand and use the derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a general point (x, y);the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x]** and for $\sin x$ and $\cos x$ **[Understand and use the second derivative as the rate of change of gradient]**; connection to convex and concave sections of curves and points of inflection
G2	**[Differentiate x^n, for rational values of n, and related constant multiples, sums and differences]** Differentiate e^{kx} and a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ and related sums, differences and constant multiples Understand and use the derivative of $\ln x$
G3	**[Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points]**, points of inflection **[Identify where functions are increasing or decreasing]**
G4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions
G5	Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only
G6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand)

H Integration

	Content
H1	**[Know and use the Fundamental Theorem of Calculus]**
H2	**[Integrate x^n (excluding n = -1), and related sums, differences and constant multiples]** Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples
H3	**[Evaluate definite integrals; use a definite integral to find the area under a curve]** and the area between two curves

H4	Understand and use integration as the limit of a sum
H5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae)
H6	Integrate using partial fractions that are linear in the denominator
H7	Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor)
H8	Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics

I Numerical methods

	Content
I1	Locate roots of $\mathrm{f}(x)=0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval of x on which $\mathrm{f}(x)$ is sufficiently well-behaved Understand how change of sign methods can fail
I2	Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1}=\mathrm{g}(x_n)$ Understand how such methods can fail
I3	Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between
I4	Use numerical methods to solve problems in context

J Vectors

	Content
J1	**[Use vectors in two dimensions]** and in three dimensions
J2	**[Calculate the magnitude and direction of a vector and convert between**

	component form and magnitude/direction form]
J3	**[Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations]**
J4	**[Understand and use position vectors; calculate the distance between two points represented by position vectors]**
J5	**[Use vectors to solve problems in pure mathematics and in context, including forces]** and kinematics

13. For sections K to O students must demonstrate the ability to use calculator technology to compute summary statistics and access probabilities from standard statistical distributions.

K Statistical sampling

	Content
K1	**[Understand and use the terms 'population' and 'sample']** **[Use samples to make informal inferences about the population]** **[Understand and use sampling techniques, including simple random sampling and opportunity sampling]** **[Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population]**

L Data presentation and interpretation

	Content
L1	**[Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency]** **[Connect to probability distributions]**
L2	**[Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded)]** **[Understand informal interpretation of correlation]** **[Understand that correlation does not imply causation]**
L3	**[Interpret measures of central tendency and variation, extending to standard deviation]** **[Be able to calculate standard deviation, including from summary**

	statistics]
L4	**[Recognise and interpret possible outliers in data sets and statistical diagrams]** **[Select or critique data presentation techniques in the context of a statistical problem]** **[Be able to clean data, including dealing with missing data, errors and outliers]**

M Probability

	Content
M1	**[Understand and use mutually exclusive and independent events when calculating probabilities]** **[Link to discrete and continuous distributions]**
M2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables Understand and use the conditional probability formula $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
M3	Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions

N Statistical distributions

	Content
N1	**[Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution]**
N2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution Link to histograms, mean, standard deviation, points of inflection and the binomial distribution
N3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate

O Statistical hypothesis testing

	Content
O1	**[Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value]**; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded)
O2	**[Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context]** **[Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis]**
O3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context

P Quantities and units in mechanics

	Content
P1	**[Understand and use fundamental quantities and units in the S.I. system: length, time, mass]** **[Understand and use derived quantities and units: velocity, acceleration, force, weight],** moment

Q Kinematics

	Content
Q1	**[Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration]**
Q2	**[Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph]**
Q3	**[Understand, use and derive the formulae for constant acceleration for motion in a straight line]**; extend to 2 dimensions using vectors
Q4	**[Use calculus in kinematics for motion in a straight line:** $v = \frac{\mathrm{d}r}{\mathrm{d}t},\ a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2},\ r = \int v\,\mathrm{d}t,\ v = \int a\,\mathrm{d}t$ **]**; extend to 2 dimensions using vectors
Q5	Model motion under gravity in a vertical plane using vectors; projectiles

R Forces and Newton's laws

	Content
R1	**[Understand the concept of a force; understand and use Newton's first law]**
R2	**[Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors)]**; extend to situations where forces need to be resolved (restricted to 2 dimensions)
R3	**[Understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy]** **[(The inverse square law for gravitation is not required and *g* may be assumed to be constant, but students should be aware that *g* is not a universal constant but depends on location)]**
R4	**[Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles]**; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces
R5	Understand and use addition of forces; resultant forces; dynamics for motion in a plane
R6	Understand and use the $\mathbf{F} \leq \mu\mathbf{R}$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics

S Moments

	Content
S1	Understand and use moments in simple static contexts

Appendix A: mathematical notation for AS and A level qualifications in mathematics and further mathematics

The tables below set out the notation that must be used by AS and A level mathematics and further mathematics specifications. Students will be expected to understand this notation without need for further explanation.

Mathematics students will not be expected to understand notation that relates only to further mathematics content. Further mathematics students will be expected to understand all notation in the list.

For further mathematics, the notation for the core content is listed under sub headings indicating 'further mathematics only'. In this subject, awarding organisations are required to include, in their specifications, content that is additional to the core content. They will therefore need to add to the notation list accordingly.

AS students will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A level content.

1	**Set Notation**	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	$\mathcal{E}$	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers

1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	$(x,\ y)$	the ordered pair x, y
1.20	$[a,\ b]$	the closed interval $\{x \in \mathbb{R} : a \le x \le b\}$
1.21	$[a,\ b)$	the interval $\{x \in \mathbb{R} : a \le x < b\}$
1.22	$(a,\ b]$	the interval $\{x \in \mathbb{R} : a < x \le b\}$
1.23	$(a,\ b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1	**Set Notation (Further Mathematics only)**	
1.24	$\mathbb{C}$	the set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \le$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \ge$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence

2.21	S_∞	sum to infinity of a sequence
3	**Operations**	
3.1	$a+b$	a plus b
3.2	$a-b$	a minus b
3.3	$a\times b,\ ab,\ a.b$	a multiplied by b
3.4	$a \div b,\ \dfrac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n\times(n-1)\times\ldots\times 2\times 1,\ n\in\mathbb{N};\ 0!=1$
3.10	$\binom{n}{r},\ {}^nC_r,\ {}_nC_r$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x\to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x,\ \delta x$	an increment of x
4.7	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x),\ \mathrm{f}''(x),\ \ldots,\ \mathrm{f}^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t

4.11	$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_\mathrm{e} x$	natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{array}{l}\sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot\end{array}\right\}$	the trigonometric functions
6.2	$\left.\begin{array}{l}\sin^{-1}, \cos^{-1}, \tan^{-1}, \\ \arcsin, \arccos, \arctan\end{array}\right\}$	the inverse trigonometric functions
6.3	$^\circ$	degrees
6.4	rad	radians
6	**Trigonometric and Hyperbolic Functions (Further Mathematics only)**	
6.5	$\left.\begin{array}{l}\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot}\end{array}\right\}$	the inverse trigonometric functions
6.6	$\left.\begin{array}{l}\sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \coth\end{array}\right\}$	the hyperbolic functions
6.7	$\left.\begin{array}{l}\sinh^{-1}, \cosh^{-1}, \tanh^{-1}, \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \coth^{-1}\end{array}\right\}$ $\left.\begin{array}{l}\operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth}\end{array}\right\}$	the inverse hyperbolic functions
7	**Complex Numbers (Further Mathematics only)**	
7.1	i, j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r\,(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\operatorname{Re}(z)$	the real part of z, $\operatorname{Re}(z) = x$
7.6	$\operatorname{Im}(z)$	the imaginary part of z, $\operatorname{Im}(z) = y$

7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta,\ -\pi < \theta \le \pi$
7.9	z^*	the complex conjugate of z, $x - \mathrm{i}y$
8	**Matrices (Further Mathematics only)**	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	Δ, det $\mathbf{M}$ or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	$\mathbf{Mr}$	Image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert \mathbf{a} \rvert$, a	the magnitude of $\mathbf{a}$
9.6	$\lvert \overrightarrow{\mathrm{AB}} \rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector

9	**Vectors (Further Mathematics only)**	
9.12	**a.b**	the scalar product of **a** and **b**
10	**Differential Equations (Further Mathematics only)**	
10.1	ω	angular speed
11	**Probability and Statistics**	
11.1	A, B, C, etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	values of observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim \mathrm{N}(0,1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0, 1)$
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance

11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	m/s, $m\ s^{-1}$	metres per second (velocity)
12.5	m/s^2, $m\ s^{-2}$	metres per second per second (acceleration)
12.6	F	Force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

Appendix B: mathematical formulae and identities

Students must be able to use the following formulae and identities for AS and A level mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a \left(x^k\right)$

Coordinate Geometry

A straight line graph, gradient m passing through $\left(x_1, y_1\right)$ has equation

$y - y_1 = m\left(x - x_1\right)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \dfrac{1}{2}ab\sin C$

$$\cos^2 A + \sin^2 A \equiv 1$$
$$\sec^2 A \equiv 1 + \tan^2 A$$
$$\text{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2\sin A\cos A$$
$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Mensuration

Circumference and Area of circle, radius r and diameter d:

$$C = 2\pi r = \pi d \qquad A = \pi r^2$$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:

$$c^2 = a^2 + b^2$$

Area of a trapezium = $\frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c,\ x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Area under a curve $= \int_a^b y \, dx \ (y \geq 0)$

Vectors

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$

Mechanics

Forces and Equilibrium

Weight = mass $\times g$

Friction: $F \le \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} \qquad a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$$

$$r = \int v\,\mathrm{d}t \qquad v = \int a\,\mathrm{d}t$$

Statistics

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{X - \mu}{\sigma}$ where $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$

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