Assessing Pupils' Progress

Focused assessment materials: Level 6

Department for Education

The National Strategies
Secondary

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department for
children, schools and families

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Acknowledgement

The National Strategies are grateful for the many contributions from teachers, consultants and students that helped to make these materials possible. Particular thanks are due to colleagues from Gloucestershire Local Authority for their contributions.

These materials are based on the APP assessment criteria and organised in the National Curriculum levels. There is a set for each of levels 4 to 8.

The focused assessment materials include for each assessment criterion:

- **Examples of what pupils should know and be able to do** so teachers have a feel for how difficult the mathematics is intended to be. These are not activities or examples that will enable an accurate assessment of work at this level. To do this, you need a broad range of evidence drawn from day-to-day teaching over a period of time; this is exemplified in the Standards files, which are provided as part of the overall APP resources.
- Some **probing questions** for teacher to use with pupils in lessons to initiate dialogue to help secure their assessment judgement.

Numbers and the number system

Examples of what pupils should know and be able to do	Probing questions
Use the equivalence of fractions, decimals and percentages to compare proportions	
Convert fraction and decimal operators to percentage operators by multiplying by 100, e.g.: • $0.45 = 0.45 \times 100\% = 45\%$ • $\frac{7}{12} = (7 \div 12) \times 100\% = 58.3\%$ (to one decimal place) Continue to use mental methods for finding percentages of quantities. Use written methods, e.g.: • using an equivalent fraction: 13% of 48; $\frac{13}{100} \times 48 = \frac{624}{100} = 6.24$ • using an equivalent decimal: 13% of 48; $0.13 \times 48 = 6.24$	Which sets of equivalent fractions, decimals and percentages do you know? From one set that you know (e.g. $\frac{1}{10} = 0.1 = 10\%$) which others can you deduce? How would you go about finding the decimal and percentage equivalents of any fraction? How would you find out which of these is closest to $\frac{1}{3}$; $\frac{10}{31}$; $\frac{20}{61}$; $\frac{30}{91}$; $\frac{50}{151}$? What links have you noticed within equivalent sets of fractions, decimals and percentages? Give me a fraction between $\frac{1}{3}$ and $\frac{1}{2}$. How did you do it? Which is it closer to? How do you know?

Calculating

Examples of what pupils should know and be able to do	Probing questions
Calculate percentages and find the outcome of a given percentage increase or decrease	
Use written methods, e.g.: • using an equivalent fraction: 19% of 67 = $\frac{19}{100} \times 67 = \frac{1273}{100}$ = 12.73 • using an equivalent decimal: 19% of 67; $0.19 \times 67 = 12.73$ • using a unitary method: 19% of 67; 1% of 67 = 0.67 so 19% of 67 = $19 \times 0.67 = 12.73$ Find the outcome of a given percentage increase or decrease, e.g.: • an increase of 15% on an original cost of £12 gives a new price of £12 × 1.15 = £13.80 • or 15% of £12 = £1.80, £12 + £1.80 = £13.80.	Talk me through how you would increase/ decrease £12 by, e.g., 15%. Can you do it in a different way? How would you find the multiplier for different percentage increases/decreases? The answer to a percentage increase question is £10. Make up an easy question. Make up a difficult question.
Divide a quantity into two or more parts in a given ratio and solve problems involving ratio and direct proportion	
Solve problems such as: • Potting compost is made from loam, peat and sand in the ratio 7:3:2 respectively. A gardener used 1.5 litres of peat to make compost. How much loam did she use? How much sand? • The angles in a triangle are in the ratio 6:5:7. Find the sizes of the three angles.	If the ratio of boys to girls in a class is 3:1, could there be exactly 30 children in the class? Why? Could there be 25 boys? Why? 5 miles is about the same as 8km. • Can you make up some conversion questions that you could answer mentally? • Can you make up some conversion questions for which you would have to use a more formal method? How would you work out the answers to these questions?

Use proportional reasoning to solve a problem, choosing the correct numbers to take as 100%, or as a whole

Use unitary methods and multiplicative methods, e.g.:

- There was a 25% discount in a sale. A boy paid £30 for a pair of jeans in the sale. What was the original price of the jeans?
- When heated, a metal bar increases in length from 1.255m to 1.262m. Calculate the percentage increase correct to one decimal place.

A recipe for fruit squash for six people is:

300g chopped oranges

1500ml lemonade

750ml orange juice

Trina made fruit squash for ten people. How many millilitres of lemonade did she use?

Jim used two litres of orange juice for the same recipe. How many people was this enough for?

Which are the key words in this problem? How do these words help you to decide what to do?

What are the important numbers? What are the important links that might help you solve the problem?

How do you decide which number represents 100% or a whole when working on problems?

Do you expect the answer to be larger or smaller? Why?

What would you estimate the answer to be? Why?

Add and subtract fractions by writing them with a common denominator, calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction

Add and subtract more complex fractions such as $\frac{11}{18} + \frac{7}{24}$, including mixed fractions.

Solve problems involving fractions, e.g.:

- In a survey of 24 pupils $\frac{1}{3}$ liked football best, $\frac{1}{4}$ liked basketball, $\frac{3}{8}$ liked athletics and the rest liked swimming. How many liked swimming?

Why are equivalent fractions important when adding or subtracting fractions?

What strategies do you use to find a common denominator when adding or subtracting fractions?

Is there only one possible common denominator?

What happens if you use a different common denominator?

Give pupils some examples of addition and subtraction of fractions with common mistakes in them. Ask them to talk you through the mistakes and how they would correct them.

How would you justify that $4 \div \frac{1}{5} = 20$? How would you use this to work out $4 \div \frac{2}{5}$? Do you expect the answer to be greater or less than 20? Why?

Algebra

Examples of what pupils should know and be able to do	Probing questions
Use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^3 + x = 20$	
Use systematic trial and improvement methods to find approximate solutions to equations, e.g.: • $a^3 + a = 20$ • $y(y + 2) = 67.89$ Use trial and improvement for equivalent problems, e.g.: • A number plus its cube is 20, what's the number? • The length of a rectangle is 2cm longer than the width. The area is 67.89cm^2. What's the width? Pupils should have opportunities to use a spreadsheet for trial and improvement methods.	How do you go about choosing a value (of x) to start? How do you use the previous outcomes to decide what to try next? How do you know when to stop? How would you improve the accuracy of your solution? Is your solution exact? Can this equation be solved using any other method? Why?
Construct and solve linear equations with integer coefficients, using an appropriate method	
Solve, e.g.: • $3c - 7 = -13$ • $4(z + 5) = 84$ • $4(b - 1) - 5(b + 1) = 0$ • $12/(x+4) = 21/(x+4)$ Construct linear equations, e.g.: • The length of a rectangle is three times its width. Its perimeter is 24cm. Find its area.	How do you decide where to start when solving a linear equation? Given a list of linear equations ask: • Which of these are easy to solve? • Which are difficult and why? • What strategies are important with the difficult ones? $6 = 2p - 8$. How many solutions does this equation have? Give me other equations with the same solution. Why do they have the same solution? How do you know? How do you go about constructing equations from information given in a problem? How do you check whether the equation works?

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT; write an expression to describe the n^{th} term of an arithmetic sequence

Generate the first five terms if, e.g.:

- you start with 100 and subtract 5 each time
- you start with 2 and double each time
- the n^{th} term is $n + 3$
- the n^{th} term is $105 – 5n$
- the n^{th} term is $2n – 0.5$

Find the n^{th} term in a sequence such as:

- 7, 12, 17, 22, 27, ...
- –12, –7, –2, 3, 8, ...
- 4, –2, –8, –14, –20, ...

Use a spreadsheet to generate tables of values and explore term-to-term and position-to-term linear relationships, e.g.:

For the n^{th} term in $3n + 7$, generate a table of values and explore the term-to-term and position-to-term relationships.

The term-to-term rule for a sequence is +2. What does that tell you about the position-to-term rule? Do you have enough information to find the rule for the n^{th} term? Why?

What do you look for in a sequence to help you to find the position-to-term (n^{th} term) rule?

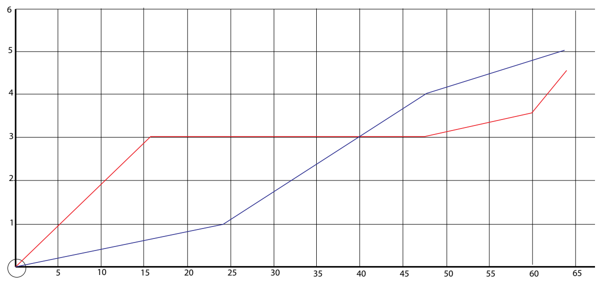
How would you go about finding the position-to-term (n^{th} term) rule for this information about a sequence?

Position	3	5	10
Term	11	19	39

Plot the graphs of linear functions, where *y* is given explicitly in terms of *x*; recognise that equations of the form $y = mx + c$ correspond to straight line graphs	
Plot the graphs of simple linear functions using all four quadrants by generating coordinate pairs or a table of values, e.g.: • $y = 2x - 3$ • $y = 5 - 4x$ Understand the gradient and intercept in $y = mx + c$, describe similarities and differences of given straight line graphs, e.g.: • $y = 2x + 4$ • $y = 2x - 3$ Without drawing the graphs, compare and contrast features of pairs of graphs such as: • $y = 3x$ • $y = 3x + 4$ • $y = x + 4$ • $y = x - 2$ • $y = 3x - 2$ • $y = -3x + 4$	How do you go about finding a set of coordinates for a straight line graph, e.g. $y = 2x + 4$? How do you decide on the range of numbers to put on the *x* and *y* axes? How do you decide on the scale you are going to use? If you increase/decrease the value of *m*, what effect does this have on the graph? What about changes to *c*? What have you noticed about the graphs of functions of the form $y = mx + c$? What are the similarities and differences?

Construct functions arising from real-life problems and plot their corresponding graphs; interpret graphs arising from real situations

The graph below shows information about a race between two animals – the hare (red) and the tortoise (blue).



- Who was ahead after two minutes?
- What happened at three minutes?
- At what time did the tortoise draw level with the hare?
- Between what times was the tortoise travelling fastest?
  - By what distance did the tortoise win the race?

What do the axes represent?

In the context of this problem does every point on the line have a meaning? Why?

What does this part of the graph represent? What does this point on the graph represent?

What sort of questions could you use your graph to answer?

For real-life problems that lead to linear functions:

- How does the gradient relate to the original problem?
- Do the intermediate points have any practical meaning?
- What's the relevance of the intercept in relation to the original problem?

Shape, space and measures

<table>
<tr><th>Examples of what pupils should know and be able to do</th><th>Probing questions</th></tr>
<tr><td colspan="2">Classify quadrilaterals by their geometric properties</td></tr>
<tr><td>Know the properties (equal and/or parallel sides, equal angles, right angles, diagonals bisected and/or at right angles, reflection and rotation symmetry) of:
• an isosceles trapezium
• a parallelogram
• a rhombus
• a kite
• an arrowhead or delta.</td><td>What properties do you need to know about a quadrilateral to be sure it is a kite; a parallelogram; a rhombus; an isosceles; a trapezium?
Can you convince me that a rhombus must be a parallelogram but a parallelogram is not necessarily a rhombus?
Why can’t a trapezium have three acute angles?
Which quadrilateral can have three acute angles?</td></tr>
<tr><td colspan="2">Solve geometrical problems using properties of angles, of parallel and intersecting lines, and of triangles and other polygons</td></tr>
<tr><td>Explain why:
• equilateral triangles, squares and regular hexagons will tessellate on their own, but other regular polygons will not
• squares and regular octagons will tessellate together.
Use and understand notation for equal sides and angles, parallel lines.
Find missing angles using properties of shapes and parallel and intersecting lines.
The angle at the vertex of a regular pentagon is 108°. Two diagonals are drawn to the same vertex to make three triangles. Calculate the size of the angles in each triangle.</td><td>Refer to given diagram with geometric notation.
• Talk me through the information that has been given to you in this diagram.
• How do you decide where to start in order to find the missing angle(s)/solve the geometrical problem?
What clues do you look for when finding a missing angle for a geometrical diagram?
What’s the minimum information you would need to be able to find all the angles in this diagram?
What clues do you look for when solving a geometrical problem? How do you decide where to start? Is it possible to solve the problem in a different way?
How would you convince somebody that the exterior angles of a polygon add up to 360°?</td></tr>
</table>

Identify alternate and corresponding angles; understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°	
Know the difference between a demonstration and a proof. Understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°.	How could you convince me that the sum of the angles of a triangle is 180°? Why are parallel lines important when proving the sum of the angles of a triangle? How does knowing the sum of the interior angles of a triangle help you to find the sum of the interior angles of a quadrilateral? Will this work for all quadrilaterals? Why?
Devise instructions for a computer to generate and transform shapes and paths	
Draw a square/hexagon/equilateral triangle using LOGO and use the instructions to compare with the exterior angles of a regular polygon. Use ICT to generate paths such as rectilinear shapes and equi-angular spirals.	What is the same/different about: • forward 90, right 90 • right 90, forward 90 Why do you need to use the exterior angles to be able to write instructions for a computer to generate a regular polygon? How would you know your instructions formed a closed shape before trying it out on a computer?
Visualise and use 2-D representations of 3-D objects	
Visualise solids from an oral description, e.g. identify the 3-D shape if: • the front and side elevations are both triangles and the plan is a square • the front and side elevations are both rectangles and the plan is a circle • the front elevation is a rectangle, the side elevation is a triangle and the plan is a rectangle. Is it possible to slice a cube so that the cross section is: • a rectangle • a triangle • a pentagon • a hexagon?	How would the 3-D shape be different if the plan was a rectangle rather than a square? Why? Are there other possible 3-D shapes? Starting from a 2-D representation of a 3-D shape: • How many faces will the 3-D shape have? How do you know? • What will be opposite this face in the 3-D shape? How do you know? • Which side will this side join to make an edge? How do you know? • How would you go about drawing the plan and elevation for the 3-D shape you could make from this net?

<table>
<tr><th colspan="2">Enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor</th></tr>
<tr><td>Construct an enlargement given the object, centre of enlargement and scale factor.

Find the centre of enlargement and/or scale factor from the object and image.</td><td>What changes when you enlarge a shape? What stays the same?

What information do you need to complete a given enlargement?

If someone has completed an enlargement how would you find the centre and the scale factor?

When doing an enlargement, what strategies do you use to make sure your enlarged shape will fit on the paper?</td></tr>
<tr><th colspan="2">Know that translations, rotations and reflections preserve length and angle and map objects onto congruent images</th></tr>
<tr><td>Find missing lengths and angles on diagrams that show an object and its image.

Match corresponding lengths and angles of object and image shapes following reflection, translation and/or rotation or a combination of these.</td><td>What changes and what stays the same when you:
translate
rotate
reflect
a shape?

When is the image congruent to the original shape? How do you know?</td></tr>
<tr><th colspan="2">Use straight edge and compasses to do standard constructions</th></tr>
<tr><td>Use straight edge and compasses to construct:
the mid-point and perpendicular bisector of a line segment
the bisector of an angle
the perpendicular from a point to a line segment
the perpendicular from a point on a line segment.
Construct triangles to scale using ruler and protractor (SAS, ASA) and using straight edge and compasses (SSS).</td><td>Why are compasses important when doing constructions?

How do the properties of a rhombus help with simple constructions such as bisecting an angle?

For which constructions is it important to keep the same compass arc (distance between the pencil and the point of your compasses)? Why?</td></tr>
</table>

Deduce and use formulae for the area of a triangle and parallelogram, and the volume of a cuboid; calculate volumes and surface areas of cuboids	
Calculate the areas of triangles and parallelograms. Suggest possible dimensions for triangles and parallelograms when the area is known. Calculate the volume and the surface area of a 3cm by 4cm by 5cm box. Find three cuboids with a volume of $24cm^3$. Find a cuboid with a surface area of $60cm^2$.	Why do you have to multiply the base by the perpendicular height to find the area of a parallelogram? The area of a triangle is $12cm^2$. What are the possible lengths of base and height? Right-angled triangles have half the area of the rectangle with the same base and height. What about non-right-angled triangles? What other formulae for the area of 2-D shapes do you know? Is there a formula for the area of every 2-D shape? How do you go about finding the volume of a cuboid? How do you go about finding the surface area of a cuboid? 'You can build a solid cuboid using a given number of identical interlocking cubes.' Is this statement always, sometimes or never true? If it is sometimes true, when is it true and when is it false? For what numbers can you only make one cuboid? For what numbers can you make several different cuboids?
Know and use the formulae for the circumference and area of a circle	
A circle has a circumference of 120cm. What is its radius? A touring cycle has wheels of diameter 70cm. How many rotations does each wheel make for every 10km travelled? A circle has a radius of 15cm. What is its area? A door is in the shape of a rectangle with a semicircular arch on top. The rectangular part is 2m high and the door is 90cm wide. What is the area of the door?	What is the minimum information you need to be able to find the circumference and area of a circle? Give pupils some work with mistakes. Ask them to identify and correct the mistakes. How would you go about finding the area of a circle if you know the circumference?

Handling data

<table>
<tr><th>Examples of what pupils should know and be able to do</th><th>Probing questions</th></tr>
<tr><td colspan="2">Design a survey or experiment to capture the necessary data from one or more sources; design, trial and if necessary refine data collection sheets; construct tables for large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables</td></tr>
<tr><td>Investigate jumping or throwing distances:
Check that the data collection sheet is designed to record all factors that may have a bearing on the distance jumped or thrown, such as age or height.
Decide the degree of accuracy needed for each factor.
Recognise that collecting too much information will slow down the experiment; too little may limit its scope.
Study the distribution of grass:
Use a quadrat or points frame to estimate the number of grass and non-grass plants growing in equal areas at regular intervals from a north-facing building. Repeat next to a south-facing building.
Increase accuracy by taking two or more independent measurements.</td><td>What was important in the design of your data collection sheet?

What changes did you make after trialling the data collection sheet and why?

Why did you choose that size of sample?

What decisions have you made about the degree of accuracy in the data you are collecting?

How will you make sense of the data you have collected? What options do you have in organising the data, including the use of two-way tables?

How did you go about choosing your class intervals? Would different class intervals make a difference to the findings? How?</td></tr>
</table>

Select, construct and modify, on paper and using ICT:

- **pie charts for categorical data**
- **bar charts and frequency diagrams for discrete and continuous data**
- **simple time graphs for time series**
- **scatter graphs**

and identify which are most useful in the context of the problem

Understand that pie charts are mainly suitable for categorical data. Draw pie charts using ICT and by hand, usually using a calculator to find angles. Draw compound bar charts with subcategories. Use frequency diagrams for continuous data and know that the divisions between bars should be labelled. Use line graphs to compare two sets of data. Use scatter graphs for continuous data with two variables, showing, e.g., weekly hours worked against hours of TV watched.	When drawing a pie chart, what information do you need to calculate the size of the angle for each category? What is discrete/continuous data? Give me some examples. How do you go about choosing class intervals when grouping data for a bar chart/frequency diagram? What's important when choosing the scale for the frequency axis? Is this graphical representation helpful in addressing the hypothesis? If not, why and what would you change? When considering a range of graphs representing the same data: • Which is the easiest to interpret? Why? • Which is most helpful in addressing the hypothesis? Why?

Find and record all possible mutually exclusive outcomes for single events and two successive events in a systematic way

Use a sample space diagram to show all outcomes when two dice are thrown together, and the scores added. One red and one white dice are both numbered 1 to 6. Both dice are thrown and the scores added. Use a sample space to show all possible outcomes.	Give me examples of mutually exclusive events. How do you go about identifying all the mutually exclusive outcomes for an experiment? What strategies do you use to make sure you have found all possible mutually exclusive outcomes for two successive events, e.g. rolling two dice? How do you know you have recorded all the possible outcomes?

Know that the sum of probabilities of all mutually exclusive outcomes is one, and use this when solving problems	
Two coins are thrown at the same time. There are four possible outcomes: head/head; head/tail; tail/head; and tail/tail. The probability of throwing head/head is ¼. What is the probability of not throwing two heads? Extend this to: • three coins • four coins • five coins.	Why is it helpful to know that the sum of probabilities of all mutually exclusive events is one? Give me an example of how you have used this when working on probability problems.
Communicate interpretations and results of a statistical survey using selected tables, graphs and diagrams in support	
Using selected tables, graphs and diagrams for support, describe the current incidence of male and female smoking in the UK, using frequency diagrams to demonstrate the peak age groups. Show how the position has changed over the past 20 years, using line graphs. Conclude that the only group of smokers on the increase is females aged 15–25.	Which of your tables/graphs/diagrams give the strongest evidence to support/reject your hypothesis? How? What conclusions can you draw from your table/graph/diagram? Convince me using the table/graph/diagram. What difference would it make if this piece of data was included? Are any of your graphs/diagrams difficult to interpret? Why?

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